4/EH–29 (iv) (Syllabus–2015)

2 0 2 1

(July)

MATHEMATICS

(Elective/Honours)

(Statics and Dynamics)

(GHS–41)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Prove that the algebraic sum of the resolved parts of any two forces acting at a point, along any direction, is equal to the resolved part of their resultant, in the same direction. 5

(b) The lines of action of two forces $P \quad Q$ and $P \quad Q$ make an angle 2 with one another and their resultant makes an angle with the bisector of the angle between them. Show that

$$P \tan \qquad Q \tan$$ 5

(c) Three like parallel forces P, Q, R act at the vertices A, B, C of the triangle ABC and are respectively proportional to a, b, c. Show that their resultant passes through the in-centre of the triangle. 5

2. *(a)* State and prove Varignon's theorem. 1+5=6

(b) P and Q are like parallel forces. An unlike parallel force ($P \quad Q$) acts in the same plane at perpendicular distances a, b respectively from the forces and between them. Find the moment of the resultant couple. 5

(c) Three forces P, Q, R act in the same sense along the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ of a triangle ABC. Show that, if their resultant passes through the in-centre, then $P \quad Q \quad R \quad 0$. 4

UNIT—II

3. *(a)* The moments of a system of coplanar forces (not in equilibrium) about three collinear points A, B, C in their plane are G_1, G_2, G_3. Prove that (with due regard to the sign)

$$G_1 \quad BC \quad G_2 \quad CA \quad G_3 \quad AB \quad 0$$ 5

(b) A uniform ladder is in equilibrium with one end resting on the ground and the other against a vertical wall; if the ground and wall be both rough, the coefficients of friction being   and   respectively and if the ladder be on the point of slipping at both ends, then show that the inclination of the ladder to the horizon is given by

$$\tan \quad \frac{1}{2}$$

4

(c) If the force which acting parallel to a rough plane of inclination   to the horizon is just sufficient to draw a weight up be n times the force which will just let it be on the point of sliding down, then show that

$$\tan \quad \frac{n \quad 1}{n \quad 1}$$

6

4. *(a)* A heavy uniform rod of length $2a$ rests in equilibrium, having one end against a smooth vertical wall and being placed upon a peg at a distance b from the wall. Show that the inclination of the rod to the horizontal is

$$\cos^{\ 1} \frac{b}{a}^{\frac{1}{3}}$$

5

(b) D, E, F are the mid-points of the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ of the triangle ABC. Masses m_1, m_2, m_3 are placed at A, B, C and masses $_1$, $_2$, $_3$ are placed at D, E, F. If the two systems have the same CG, then prove that

$$\frac{m_1}{\ _2 \quad _3} \qquad \frac{m_2}{\ _3 \quad _1} \qquad \frac{m_3}{\ _1 \quad _2}$$

5

(c) At each of $n \quad 1$ of the angular points of a regular polygon of n sides equal particles are placed. Show that the distance of the CG from the circumcentre of the polygon is $\frac{r}{n \quad 1}$, where r is the circumradius. 5

UNIT—III

5. *(a)* A particle starts from rest under an acceleration k^2x directed towards a fixed point and after time t_1 another particle starts from the same position under the same acceleration. Show that the particle will collide at time $\frac{\ }{k} \ \frac{t_1}{2}$ after the start of the first particle, provided $t_1 \ \frac{2}{k}$.

5

(b) A particle moves along the axis of x starting from rest at x a. For an interval t_1 from the beginning of motion the acceleration is x, for a subsequent time t_2 the acceleration is x and at the end of this interval the particle is at the origin. Prove that $\tan(\sqrt{\ }t_1)\tanh(\sqrt{\ }t_2)$ 1. 6

(c) A particle moves towards a centre of attraction starting from rest at a distance a from the centre. If its velocity, when at any distance x from the centre, varies as $\sqrt{\frac{a^2 \quad x^2}{x^2}}$, then find the law of force. 4

6. *(a)* Three equal spheres are in a straight-line on a table and one of these moves towards the other two which are at rest and not in contact. If e $\frac{1}{2}$, then show that three impacts will take place and the ultimate speeds of the spheres will be in the ratio 13 : 15 : 36. 5

(b) Two equal spheres of elasticity e impinge, having before impact velocities u_1, v_1 in the directions of common normal and u_2, v_2 perpendicular to this normal. If after impact the spheres move perpendicularly to each other, then show that $(u_1 \quad v_1)^2 \quad 4u_2v_2 \quad e^2(u_1 \quad v_1)^2$. 5

(c) *(i)* State the laws of elastic impact. 1+1=2

(ii) A heavy elastic ball drops from the ceiling of a room and after rebounding twice from the floor reaches a height equal to one-half of that of the ceiling. Show that $2e^4$ 1. 3

UNIT—IV

7. *(a)* A projectile aimed at a mark, which is in the horizontal plane through the point of projection, falls a metre short of it when the elevation is and goes b metre too far when the elevation is . Show that if the velocity of projection be the same in all the cases, the proper elevation is

$$\frac{1}{2}\sin^{1}\frac{a\sin 2 \quad b\sin 2}{a \quad b}$$ 5

(b) A motorboat, of weight w, is moving at 25 m/sec when the motor is stopped. If the resistance to the motion is then $k\sqrt{v}$ and the boat comes to rest after having moved 100 metres, find the time it took to cover these 100 metres. 4

(c) A body is projected at an angle to the horizon, so as just to clear two walls of equal height a at a distance $2a$ from each other. Show that the range is equal to $2a\cot(\frac{}{2})$. 6

8. *(a)* Show that a particle, projected vertically upwards with a velocity U in a medium whose resistance varies as the square of the velocity will return to the point of projection with velocity

$$v \quad \frac{UV}{\sqrt{U^2 \quad V^2}} \text{ after a time}$$

$$\frac{V}{g} \tan^{\ 1} \frac{U}{V} \quad \tanh^{\ 1} \frac{v}{V}$$

where V is the terminal velocity. 9

(b) Particles are projected simultaneously in the same vertical plane from the same point. Show that the locus of the foci of all the trajectories is a parabola, when for each trajectory there is the same *(i)* horizontal velocity, *(ii)* initial vertical velocity and *(iii)* time of flight. 2+2+2=6

UNIT—V

9. *(a)* A particle is acted upon by a force parallel to the axis of y whose acceleration (always towards the x-axis) is $\frac{}{y^2}$. It is projected parallel to the axis of x with velocity $\sqrt{\frac{2}{a}}$ at the point $(0, a)$. Prove that it will describe a cycloid. 5

(b) A particle is hanging from a fixed point by a light cord one metre long and is started moving with an initial horizontal velocity such that the cord slackens when the particle is $\frac{5}{3}$ metres above the lowest point. Show that it will rise further through $\frac{5}{27}$ metres. 4

(c) A gun is mounted on a gun-carriage movable on a smooth horizontal plane and the gun is elevated at an angle to the horizon. A shot is fired and leaves the gun in a direction inclined at an angle to the horizon. If the mass of the gun and the carriage be n times that of the shot, then show that

$$\tan \quad 1 \ \frac{1}{n} \ \tan$$

6

10. *(a)* A particle slides down from rest the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half. 5

(b) A particle moves in catenary S $C \tan$ and the direction of its acceleration at any point makes equal angles with the tangent and the normal to the path at that point. If the speed at the vertex (where 0) be u, then show that the velocity and the acceleration at any point are given by ue and

$$\frac{\sqrt{2}}{c} u^2 e^2 \quad \cos^2$$ 5

(c) The earth's attraction on a particle varies inversely as the square of its distance from the earth's centre. A particle whose weight on the surface of the earth is W, falls to the surface of the earth from a height $5a$ above it. Show that the work done by the earth's attraction is $\frac{5aW}{6}$, where a is the radius of the earth. 5

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