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(July)

MATHEMATICS

(Honours)

(Discrete Mathematics)

(HOPT–62 : OP5)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Show that every sequence of distinct $(n^2 \quad 1)$ real numbers contains a subsequence of length $(n \quad 1)$ which is strictly increasing or strictly decreasing. 5

(b) Find the solution of the recurrence relation $a_n \quad 5a_{n \quad 1} \quad 3$ for $n \quad 2$, $a_1 \quad 2$. 5

(c) Show that $2^n \quad 1$ is divisible by 3 for all odd integers n. 5

2. *(a)* Find in how many ways an odd number of objects can be chosen from n objects. 5

(b) Let $a, b, \quad \mathbb{N}$ be coprimes. Show that $ax \quad by \quad 1$ for some $x, y \quad \mathbb{N}$. 5

(c) Solve the equation

$$a_r \quad 5a_{r \quad 1} \quad 6a_{r \quad 2} \quad 2^r \quad 2, \; r \quad 2$$

with initial condition $a_0 \quad 1$, $a_1 \quad 1$. 5

UNIT—II

3. *(a)* Find the Hasse diagram of the partially ordered set $(A, \quad)$ if $A \quad \{2, 3, 4, 6, 8, 12, 16, 48\}$ and the partial order on A is defined by $x \quad y$ if and only if x divides y; $x, y \quad A$. 4

(b) Let (X, R) be a partially ordered set. Show that the dual $(X, \overline{R})$ of (X, R) is also a partially ordered set. 5

(c) Let $(L, \quad)$ be a lattice and $a, b, c, d \quad L$. Show that—

(i) $a \quad b, c \quad d \quad a \quad c \quad b \quad d$

(ii) $a \quad b, c \quad d \quad a \quad c \quad b \quad d$ 6

4. *(a)* Let L, K be two lattices and $f : L \quad K$ be an isomorphism. Show that $a \quad b$ if and only if $f(a) \quad f(b)$, $a, b \quad L$. 5

(b) Let L, K be two lattices. Define on L K as follows : for (a_1, b_1), (a_2, b_2) L K, (a_1, b_1) (a_2, b_2) if and only if a_1 b_1, and a_2 b_2. Show that $(L$ K, $)$ is a lattice. 6

(c) Give an example with justification of two isomorphic lattices L and K with L K. 4

UNIT—III

5. *(a)* Show that in a bounded distributive lattice, complements are unique. 4

(b) Using Karnaugh map, simplify the Boolean function

$f(x, y, z)$ xyz xyz xy z x yz 6

(c) State and prove a necessary condition for a non-empty subset of a Boolean algebra to be a subalgebra. 5

6. *(a)* If a, b, c are elements of a Boolean algebra B, show that

a $(a$ c $b)$ $(a$ a $c)$ b

for any a B. 5

(b) Show that the dual of a modular lattice is a modular lattice. 5

(c) Draw the bridge circuit for the Boolean function

f xw y uv $(xz$ $y)$ $(zw$ $uv)$ 5

UNIT—IV

7. *(a)* Show that a graph is bipartite if it has no odd cycle. 5

(b) Show that there is no graph having 5 vertices whose degrees of the vertices are 1, 2, 2, 4 and 5 respectively. 5

(c) Show that a connected graph is Eulerian if all its vertices have even degree. 5

8. *(a)* Show that every graph having degree of each vertex even decomposes into cycles. 5

(b) Find the least number of vertices needed to construct a complete graph with at least 1000 edges. 4

(c) Show that a graph with n vertices is Hamiltonian if the sum of the degrees of each pair of non-adjacent vertices is greater than or equal to $(n$ $1)$. 6

UNIT—V

9. *(a)* Show that an edge of a connected graph is a bridge if and only if there exist vertices u and v such that every path between these two vertices contains this edge. 5

(b) Give examples with justification of the following : 2+2=4

(i) A graph with 2 cut vertices but no edge cut

(ii) A graph having connectivity 2

(c) Let *G* be a connected graph with at least two vertices. If the number of edges in *G* is less than the number of vertices, show that *G* has a vertex of degree one. 6

10. *(a)* Show that a graph is a tree if and only if it is connected and every edge in it is a bridge. 5

(b) Show that every cut set in a connected graph contains at least one branch of each spanning tree of the graph. 5

(c) Show that a graph *G* is 2-connected if and only if *G* is connected with at least three vertices but no cut vertex. 5

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