3/EH–29 (iii) (Syllabus–2019)

2022

(November)

MATHEMATICS

(Elective/Honours)

(GHS–31)

(Statics and Calculus—II)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. (a) The angle of inclination between two forces P and Q is θ. If P and Q be interchanged in position, then show that the resultant will be turned through an angle ϕ, where

$$\tan\frac{\phi}{2} = \frac{P-Q}{P+Q}\tan\frac{\theta}{2}$$ 5

(b) The forces P, Q, R acting along $\overrightarrow{IA}$, $\overrightarrow{IB}$, $\overrightarrow{IC}$, where I is the incentre of the triangle ABC, are in equilibrium, show that

$$P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$$ 5

(c) P, Q are like parallel forces. If P is moved parallel to itself through a distance x, then show that the resultant of P, Q moves through a distance

$$\frac{Px}{P+Q}$$ 5

2. (a) Two forces P and Q acting at a point have got a resultant R, if Q be doubled, R is doubled. Again, if Q be reversed in direction, then also R is doubled. Show that

$$P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$$ 5

(b) Show that the moment of a force about a point is equal to the algebraic sum of the moments of its components about that point. 5

(c) $ABCD$ is a rectangle such that $AB = CD = a$ and $BC = DA = b$. The force P acts along $\vec{AD}$ and $\vec{CB}$ and force Q acts along $\vec{AB}$ and $\vec{CD}$. Prove that the perpendicular distance between the resultant of the forces P, Q at A and the resultant of the forces P, Q at C is

$$\frac{P \cdot a - Q \cdot b}{\sqrt{P^2 + Q^2}}$$ 5

UNIT—II

3. (a) The forces proportional to 1, 2, 3, 4 act along the sides $\vec{AB}$, $\vec{BC}$, $\vec{AD}$, $\vec{DC}$ respectively of a square $ABCD$, the length of whose each side is 2 m. Find the magnitude and line of action of the resultant. 2+3=5

(b) The forces P, Q, R, S act along the sides $\vec{AB}$, $\vec{BC}$, $\vec{CD}$, $\vec{DA}$ of the cyclic quadrilateral $ABCD$, taken in order, where A and B are the extremities of a diameter. If they are in equilibrium, then prove that

$$R^2 = P^2 + Q^2 + S^2 + \frac{2PQS}{R}$$ 5

(c) A body of weight W can just be sustained on a rough inclined plane by a force P, and just dragged up the plane by a force Q. P and Q both acting up the line of greatest slope. Show that the coefficient of friction is

$$\frac{Q-P}{\sqrt{4W^2 - (P+Q)^2}}$$ 5

4. (a) A heavy uniform rod of length $2a$ rests in equilibrium, having one end against a smooth vertical wall, and being placed upon a peg at a distance b from the wall.

Show that the inclination of the rod to the horizontal is

$$\cos^{-1}\left(\frac{b}{a}\right)^{1/3}$$ 5

(b) A uniform ladder rests with one end on a rough horizontal ground and the other against a rough vertical wall. The coefficients of friction at lower and upper ends are $\frac{3}{7}$ and $\frac{1}{3}$ respectively. Determine the angle which the ladder makes with the ground when it is about to slip. 5

(c) Find the centre of gravity of a triangle formed by three uniform rods. 5

Unit—III

5. (a) Prove that if a sequence converges, then its limit is unique. 5

(b) Show that the sequence $\{x_n\}$, where

$$x_n = \frac{3n+1}{n+2}$$

is strictly increasing. Is the sequence convergent? Justify your answer. Also find its limit. 2+1+2=5

(c) Define Cauchy sequence. Is the sequence $\{n^2\}$ a Cauchy sequence? Justify your answer. 1+4=5

6. (a) Test the convergence of the following series (any *two*) : 3+3=6

(i) $1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}+\cdots$

(ii) $\sum_{n=2}^{\infty}\frac{1}{\sqrt{n(n-1)}}$

(iii) $\sum_{n=1}^{\infty}\frac{n}{3^{n-1}}$

(iv) $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$

(b) What is an absolute convergent series? Test the absolute convergence of the series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$$ 1+3=4

(c) Determine the region of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{(x-3)^n}{n\cdot 3^n}$$ 5

UNIT—IV

7. (a) State and prove Lagrange's mean value theorem of differential calculus. 1+4=5

(b) Verify Rolle's theorem for the function

$$f(x) = x^2 - 5x + 6$$

in the interval [1, 4]. 4

(c) Show that $x^{1/x}$ $(x > 0)$ has a maximum at $x = e$ and deduce that $e^{\pi} > \pi^{e}$. 3+1=4

(d) Show that the curve $y^3 = 8x^2$ is concave to the foot of the ordinate everywhere except at the origin. 2

8. (a) Show that

$$\lim_{(x,\, y)\to(0,\, 0)} \frac{2xy}{x^2 + y^2}$$

does not exist. 2

(b) If $V = f(u)$, u being a homogeneous function of degree n in x and y, show that

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = nu\frac{\partial V}{\partial u}$$ 5

(c) Show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

if $u = \log(x^2 + y^2)$. 4

(d) If

$$u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$$

Show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$. 4

UNIT—V

9. (a) State and prove the fundamental theorem of integral calculus. 1+4=5

(b) Use Newton's method to find the second approximation of the root of the equation $x^4 - 12x + 7 = 0$ whose first approximation is 2. 5

(c) Expand $f(x) = \cos x$ in a finite series in powers of x with the remainder in Cauchy's form. 5

10. *(a)* Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum. 5

(b) Find the length of the circumference of the circle $x^2 + y^2 = a^2$. 5

(c) Evaluate

$$\iint (x^2 + y^2)\, dx\, dy$$

over the region bounded by $x \geq 0$, $y \geq 0$ and $x + y \leq 1$. 5

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