2022

(November)

MATHEMATICS

(Honours)

(H-54)

(Advanced Dynamics)

Marks : 45

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **three** questions, choosing **one** from each Unit

UNIT—I

1. (a) A particle describes the curve $r^n = a^n \cos n\theta$ under a force P to the pole. Find the law of force. 4

   (b) A particle moves under a force

   $$m\mu\{3au^4 - 2(a^2 - b^2)u^5\},\ a > b$$

   and is projected from an apse at a distance $a+b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit is $r = a + b\cos\theta$. 7

(c) If v_1 and v_2 are the linear velocities of a planet when it is respectively nearest and farthest from the sun, prove that $(1-e)v_1 = (1+e)v_2$ where e is the eccentricity. 4

2. (a) A particle slides in a vertical plane down a rough cycloidal arc whose axis is vertical and vertex downwards starting from a point where the tangent makes an angle θ with the horizon and coming to rest at the vertex. Show that

$$\mu e^{\mu\theta} = \sin\theta - \mu\cos\theta$$ 9

(b) A particle is describing an ellipse of eccentricity e about a centre of force at a focus. Prove with usual notation

$$v^2 = \mu\left[\frac{2}{r} - \frac{1}{a}\right]$$

$$h^2 = \mu a(1-e^2)$$ 6

UNIT—II

3. (a) Find the moment of inertia of a rectangular parallelopiped of edges $2a$, $2b$, $2c$ about an axis through the centre and parallel to one of its edges. 5

(b) Show that the momental ellipsoid at the centre of an ellipsoid is

$$(b^2+c^2)x^2+(c^2+a^2)y^2+(a^2+b^2)z^2 = \text{constant}$$ 7

(c) State and prove the parallel axis theorem. 1+2

4. (a) A uniform rectangular lamina $ABCD$ is such that $AB = 2a$, $BC = 2b$. Find the directions of the principal axes at A. 6

(b) A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to other end in time

$$\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$$

where a is the length of the plank. 9

UNIT—III

5. (a) Two unequal masses M and M' rest on two rough planes inclined at angles α and β to the horizon. They are connected by a fine string passing over a small pulley of mass m and radius a, which is placed at the common vertex

of the two planes. Show that the acceleration of either mass is

$$\frac{g[M(\sin\alpha - \mu\cos\alpha) - M'(\sin\beta + \mu'\cos\beta)]}{M + M' + m\dfrac{k^2}{a^2}}$$

where μ and μ' are the coefficients of friction, k is the radius of gyration of the pulley about its axis and M the mass which moves downwards. 10

(b) Find the length of the simple equivalent pendulum of a circular disc with axis tangent to the plane of the disc. 5

6. *(a)* A uniform triangular lamina can oscillate in its own plane about the angle A. Prove that the length of simple equivalent pendulum is

$$\frac{3(b^2 + c^2) - a^2}{4\sqrt{(2b^2 + 2c^2 - a^2)}}$$

the axis through A being horizontal. 8

(b) A cylinder rolls down a smooth plane whose inclination to the horizontal is α, unwrapping as it goes, a fine string fixed to the highest point of the plane, find its acceleration and tension of the string. 7

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