2022

(November)

MATHEMATICS

(Honours)

(H–53)

(Differential Equations)

Marks : 30

Time : 2 hours

The figures in the margin indicate full marks for the questions

Answer **two** questions, taking **one** from each Unit

UNIT—I

1. *(a)* By using normal form, solve the following equation : 5

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$$

(b) Solve the following equation : 4

$$yz(y+z)dx + zx(x+z)dy + xy(x+y)dz = 0$$

(c) Use the method of variation of parameters to solve the following equation : 6

$$y'' - y' - 2y = 4x^2$$

2. *(a)* Transform the equation

$$x^6y'' + 3x^5y' + a^2y = \frac{1}{x^2}$$

by changing the independent variable and hence solve it. 6

(b) Solve the following simultaneous equations : 5

$$\frac{dx}{dt} = x - 2y$$

$$\frac{dy}{dt} = 5x + 3y$$

(c) Solve the following : 4

$$\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$$

UNIT—II

(In the following questions, p and q denote $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$)

3. *(a)* Solve the following : 5

$$x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$$

(b) Form a partial differential equation by eliminating f from

$$f(x + y + z,\ x^2 + y^2 - z^2) = 0$$ 4

(c) Find the singular integral of

$$z = px + qy + c\sqrt{1 + p^2 + q^2}$$ 6

4. *(a)* Find the differential equation of the integral surface of

$$2y(z-3)p + (2x - z)q = y(2x - 3)$$

which passes through the circle $z = 0$, $x^2 + y^2 = 2x$. 5

(b) Find the complete integral by Charpit's method of $xp + 3yq = 2(z - x^2q^2)$. 5

(c) Find the complete integral of

$$p^2 + q^2 = (x^2 + y^2)z$$ 5

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