5/H–24 (v) (Syllabus–2020)

2022

(November)

PHYSICS

(Honours)

[PHY–05 (T–A)]

(Mathematical Physics—II, Quantum Mechanics—II)

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question No. **1** which is compulsory and *any* **four** from the rest

1. *(a)* Using beta and gamma functions, evaluate the integral $\int_0^\infty \frac{1}{(1+x)^5} dx$. 4

(b) Find the value of $\Gamma(9/4)$. 2

(c) Is the matrix $\begin{bmatrix} 1 & 2-i \\ 2+i & 0 \end{bmatrix}$ Hermitian? 2

2. (a) Obtain the Rodrigues' formula for the Legendre polynomial $P_n(x)$ and hence find the values of $P_0(x)$ and $P_1(x)$. 5+1+1=7

(b) State and prove Cauchy's integral formula. 1+4=5

3. (a) Diagonalize the matrix $\begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}$. 5

(b) Prove Cauchy's residue theorem. Use it to evaluate the integral

$$I = \int_0^{2\pi} \frac{d\theta}{5+\cos\theta}$$ 4+3=7

4. (a) Prove that $\Gamma(n+1) = n\Gamma(n)$ and hence obtain the value of $\Gamma\left(\frac{1}{2}\right)$. 2+3=5

(b) What are covariant and contravariant tensors? Write the transformation equation of a contravariant tensor. 2+1=3

(c) Show that a square matrix can be expressed as a sum of a symmetric and an anti-symmetric matrix. 4

5. (a) What do you mean by eigenvalue, eigen-vector and eigen equation of an operator? Prove that eigenvalues of a Hermitian operator are real. 1+1+1+4=7

(b) Obtain the expression for the L_Z-component of angular momentum operator in spherical polar coordinates. 5

6. (a) State and prove the Ehrenfest theorem. 7

(b) Calculate $\langle p^2 \rangle$ for the wave function

$$\psi(x) = \begin{cases} \sqrt{2/l}\,\sin \pi x/l, & 0 < |x| < l \\ 0 & , \ |x| > l \end{cases}$$ 5

7. (a) A particle of mass m is moving in a one-dimensional potential given by

$$V(X) = \begin{cases} 0, & \text{for } x < 0 \\ V_0, & \text{for } x \geq 0 \end{cases}$$

If the energy E of the incident particle is greater than V_0, then calculate the coefficient of reflection and transmission. 8

(b) Obtain these commutation relations for the Pauli spin matrices : 2+2=4

(i) $[\hat{\sigma}_x, \hat{\sigma}_y] = 2i\hat{\sigma}_z$

(ii) $[\hat{\sigma}^2, \hat{\sigma}_z] = 0$

8. *(a)* Write Schrödinger equation for the hydrogen atom in spherical polar coordinates. Separate the radial and angular parts, and solve the radial equation to show that energy of the hydrogen atom is quantized. 1+1+6=8

(b) Obtain the Heisenberg uncertainty principle by operator method. 4

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