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(May/June)

MATHEMATICS

(Honours)

(Operation Research)

(HOP–2)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Old hens can be bought for ₹ 2 each but young ones cost ₹ 5 each. The old hens lay 3 eggs per week and the young ones 5 eggs per week, each being worth 30 paise. A hen costs ₹ 1 per week to feed. If I have only ₹ 80 to spend for hens, how many of each kind should I buy to give a profit of more than ₹ 6 per week, assuming that I cannot house more than 20 hens? Write a mathematical model of the problem. 7

(b) Solve the following LPP graphically : 8

Minimize $Z = -x_1 + 2x_2$

subject to the constraints

$$-x_1 + 3x_2 \le 10$$
$$x_1 + x_2 \le 6$$
$$x_1 - x_2 \le 2$$
$$x_1 \ge 0,\ x_2 \ge 0$$

2\. (a) Apply simplex method to solve the following LPP : 9

Maximize $Z = 4x_1 + x_2$

subject to

$$x_1 + 2x_2 \le 10$$
$$4x_1 + 3x_2 \le 24$$
$$x_1 \ge 0,\ x_2 \ge 0$$

(b) Write the dual of the LPP : 6

Minimize $Z = 4x_1 + 6x_2 + 18x_3$

subject to the constraints

$$x_1 + 3x_2 \ge 3$$
$$x_2 + 2x_3 \ge 5$$
and $$x_1, x_2, x_3 \ge 0$$

UNIT—II

3\. (a) Explain briefly the following : 3+2=5

(i) Duality in transportation problem

(ii) Loops in transportation problem table

(b) Solve the following transportation problem : 10

Destinations

	D_1	D_2	D_3	D_4	*Supply*
S_1	7	10	14	8	30
Sources S_2	7	11	12	6	40
S_3	5	8	15	9	30
Demand	20	20	25	35	

4\. (a) Prove that a necessary and sufficient condition for the existence of a feasible solution to the general transportation problem is that

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \lambda \text{(say)}$$

where a_i = quantity of commodity available at origin i

b_j = quantity of commodity needed at destination j 7

(b) Obtain an initial basic feasible solution to the following transportation problem using the least-cost method : 8

	D_1	D_2	D_3	D_4	Capacity
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Demand	4	6	8	6	

UNIT—III

5. (a) Give the mathematical formulation of an assignment problem. 5

(b) A departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below :

Tasks	Men			
	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a man, so as to minimize the total man-hours? 10

6. (a) What do you understand by a Markov chain? State the conditions satisfied by a finite-state Markov chain. 2+4=6

(b) On January 1 (this year), Bakery A had 40% of its local market share while the other two Bakeries B and C had 40% and 20% respectively of the market share. Based upon a study by a marketing research firm, the following facts were compiled. Bakery A retains 90% of its customers while gaining 5% of competitor B's customers and 10% of C's customers. Bakery B retains 85% of its customers while gaining 5% of A's customers and 7% of C's customers. Bakery C retains 83% of its customers and gains 5% of A's customers and 10% of B's customers. What will each firm's share be on January1, next year, and what will each firm's market share be at equilibrium? 9

UNIT—IV

7. (a) Describe briefly the following : 2×3=6

(i) Optimum strategy

(ii) Saddle point

(iii) Pay-off matrix

(b) Consider the game G with the following pay-off matrix :

Player B

		B_1	B_2
Player A	A_1	2	6
	A_2	−2	λ

(i) Show that G is strictly determinable, whatever λ may be.

(ii) Determine the value of G and also find the optimum strategies for player A and B. 2+2=4

(c) Solve the following 2×2 game whose pay-off matrix is given below : 5

Player B

		B_1	B_2
Player A	A_1	4	−4
	A_2	−4	4

8. *(a)* Find the ranges of a and b which will render the entry (2, 2) a saddle point for the following game : 5

Player B

$$\text{Player } A \begin{bmatrix} 2 & 4 & 5 \\ 10 & 7 & b \\ 4 & a & 6 \end{bmatrix}$$

(b) Solve the game whose pay-off matrix is given below :

Player B

$$\text{Player } A \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{matrix} B_1 & B_2 & B_3 & B_4 & B_5 \end{matrix} \\ \begin{bmatrix} -2 & 0 & 0 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & -3 & 0 & -2 & 6 \\ 5 & 3 & -4 & 2 & -6 \end{bmatrix} \end{array}$$

Is the game fair? 5

(c) Two players P and Q simultaneously show 2 or 3 fingers. If the sum of the fingers shown is even, then P wins the sum from Q, if the sum is odd, then P loses the sum to Q. Find the optimum strategies for the players and to whom the game is favourable. 5

UNIT—V

9. *(a)* Solve the following game graphically : 6

Player B

$$\text{Player } A \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix}$$

(b) Solve the following game using the principle of dominance : 9

Player B

Player A	I	II	III	IV	V	VI
1	4	2	0	2	1	1
2	4	3	1	3	2	2
3	4	3	7	−5	1	2
4	4	3	4	−1	2	2
5	4	3	3	−2	2	2

10. (a) Solve the following game by linear programming technique : 10

Player B

$$\text{Player } A \begin{bmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{bmatrix}$$

(b) Use matrix Oddment method to solve the following 3×3 game : 5

Player B

$$\text{Player } A \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

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