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(May/June)

STATISTICS

(Honours)

(Statistical Inference)

[STH–61 (TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Define minimum variance unbiased estimator. A random sample X_1, X_2, X_3 and X_4 is taken from $N(\mu, \sigma^2)$. Consider the following estimators of the mean μ :

$$T_1 = \frac{X_1 + X_2 + X_3 + X_4}{4} \text{ and}$$

$$T_2 = \frac{X_1 + 2X_2 + 3X_3 + X_4}{7}$$

Which estimator should be preferred?

1+5=6

(b) Define sufficient statistic. Let $X_1, X_2, \cdots, X_n$ be a random sample from a normal distribution with mean μ and variance $\sigma^2 = 1$. Show that

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

the sample mean is a sufficient statistic for the population mean μ. 1+5=6

2. *(a)* Define consistent estimator. Show that the proportion of success in a series of n trials with constant probability of success p for each trial is a consistent estimator of the population proportion of success P. 1+5=6

(b) Let X and Y be random variables such that $E(Y) = \mu$ and $\text{var}(Y) = \sigma_Y^2 > 0$. Also let $E(Y \mid X = x) = \phi(x)$. Then prove that—

(i) $E[\phi(x)] = \mu$

(ii) $\text{var}[Y] \geq V[\phi(x)]$ 6

UNIT—II

3. *(a)* Define likelihood function. State the regularity conditions under which the maximum likelihood estimators are consistent. 1+4=5

(b) Let $X_1, X_2, \cdots, X_n$ be a random sample of n observations from a Bernoulli population with parameter p. Find the maximum likelihood estimators of *(i)* p and *(ii)* p^2. 6

4. *(a)* Explain the method of minimum chi-square for estimating parameters. Where is it used? 4+1=5

(b) Find the 95% confidence limits for the parameter λ of the Poisson distribution. 6

UNIT—III

5. *(a)* Explain what is meant by 'testing of hypothesis'. Discuss the two types of errors that arise in the testing of hypothesis. 2+3=5

(b) In order to test whether a coin is perfect, the coin is tossed 5 times. The null hypothesis of perfectness is rejected if more than 4 heads is obtained. What is the probability of type I error? Find the probability of type II error when corresponding probability of head is 0·2. Also find the power of the test. 2+2+2=6

6. *(a)* Explain the following terms : 2×3=6

(i) Size of a test

(ii) Best critical region

(iii) Most powerful test

(b) Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Obtain the most powerful test for testing $H_0 : \mu = \mu_0$ against $H_0 : \mu = \mu_1 (\mu_1 > \mu_0)$, where $\sigma^2 = 1$. 5

UNIT—IV

7. (a) State and prove Neyman-Pearson lemma for testing a simple null hypothesis against a simple alternative. 6

(b) Describe the likelihood ratio test and state its important properties. 3+2=5

8. (a) Give the sequential probability ratio test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$ in the sampling from a normal distribution. 7

(b) Define OC and ASN functions. 4

UNIT—V

9. (a) Differentiate between large sample and small sample tests and discuss their consequences in testing of hypothesis problems. How does the central limit theorem help in drawing the large sample tests? 2+2+2=6

(b) Using central limit theorem, obtain the large sample test for the difference of two binomial proportions. Also write down the confidence intervals for difference of proportions. 4+1=5

10. (a) Given any two independent populations, derive the large sample tests and hence the related confidence interval for difference of means. 6

(b) Obtain the test of significance for single mean from normal population with mean μ and variance σ^2. Hence write down the related confidence interval. 4+1=5

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