2023

(May/June)

STATISTICS

(Honours)

(Survey Sampling and Nonparametric Inference)

[STH–62 (TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* What is circular systematic sampling? Write the procedure of drawing such type of samples. How does it differ from linear systematic sampling? 5

(b) From a population of size $N = nk$ if a systematic sample of size n is drawn, then show that the variance of the mean of a systematic sample is

$$V(\bar{y}_{sys}) = \frac{N-1}{N} S^2 - \frac{k(n-1)}{N} S^2_{wsy}$$

where

$$S^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij}-\bar{Y})^2 \text{ and}$$

$$S^2_{wsy} = \frac{1}{k(n-1)}\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij}-\bar{y}_i)^2$$

7

2. *(a)* In simple random sampling without replacement, show that the regression estimator of mean is a biased estimator. 4

(b) Prove that in simple random sampling without replacement, for large n, an approximation to the variance of $\hat{R}$ is given by

$$V(\hat{R}) = \frac{1-f}{n\bar{X}^2}\frac{1}{N-1}\sum_{i=1}^{N}(y_i - Rx_i)^2$$

where $f = \frac{n}{N}$ is the sampling fraction. 8

UNIT—II

3. *(a)* What do you mean by cluster sampling? What are the advantages of cluster sampling? 4

(b) In cluster sampling with equal clusters, prove that mean of cluster means in the sample, i.e., $\bar{y}_c$ is an unbiased estimator of the population mean $\bar{Y}$ and its sampling variance is given by

$$V(\bar{y}) = \frac{1-f}{n}S_b^2 \approx \frac{1-f}{nM}S^2[1+(M-1)\rho]$$

where ρ is the intra-cluster correlation coefficient of observations and M is the cluster size. 7

4. *(a)* In cluster sampling with equal size cluster, under simple random sampling with replacement, obtain the estimate of population total along with its variance. 7

(b) Write a note on relative efficiency of cluster sampling with respect to simple random sampling. 4

UNIT—III

5. *(a)* Briefly describe two-stage sampling and give an example of it. 4

(b) In two-stage sampling, if first stage units are selected under SRSWR scheme and second stage units are selected under SRSWOR scheme, then

show that the sample mean is an unbiased estimator of population mean, i.e., $E(\bar{y}) = \bar{Y}$ and its sampling variance is given by

$$V(\bar{y}) = \frac{S_b^2}{n} + (1 - f_1)\frac{S_w^2}{nm} \qquad 7$$

6. In two-stage sampling, if n first stage units are selected and from each selected first stage unit m second stage units are selected under SRSWOR scheme, then prove that the estimator of variance of sample mean $\bar{y}$ is given by

$$v(\bar{y}) = \frac{1-f}{n} s_b^2 + \frac{f(1-f_1)}{nm} s_w^2$$

where

$$f = \frac{n}{N},\ f_1 = \frac{m}{M},\ s_b^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\bar{y}_i - \bar{y})^2 \text{ and}$$

$$s_w^2 = \frac{1}{n(m-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}(y_{ij} - \bar{y}_i)^2 \qquad 11$$

UNIT—IV

7. *(a)* What is order statistics? What purpose does it serve? 4

(b) Obtain the distribution of the rth order statistics from uniform distribution. 7

8. Obtain the joint p.d.f. of two-order statistics and hence obtain the distribution of range. 11

UNIT—V

9. *(a)* Distinguish between parametric and non-parametric approaches to the theory of statistical inference. 5

(b) Write a note on Mann-Whitney test. 6

10. Write short notes on the following : 5½×2=11

(a) Wilcoxon signed-rank test

(b) Kolmogorov-Smirnov one-sample test

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