2/EH–29 (ii) (Syllabus–2015)

2016

(April)

MATHEMATICS

(Elective/Honours)

SECOND PAPER

(Geometry and Vector Calculus)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each unit

UNIT—I

1. *(a)* Show that the equation

$$4x^2 + 12xy + 9y^2 + 8x + 12y = 0$$

represents a pair of parallel lines and find the distance between them. 5

(b) If the straight lines represented by the equation

$$x^2(\tan^2\phi + \cos^2\phi) - 2xy\tan\phi + y^2\sin^2\phi = 0$$

make angles α and β with the axis of x, then show that $\tan\alpha - \tan\beta = 2$. 5

(c) (i) Find the equation of the diameter of the conic $4x^2+6xy-5y^2=1$ conjugate to the diameter $y=2x$.

(ii) Find the asymptotes of the hyperbola $xy+4x+3y+5=0$. 3+2=5

2. (a) Find the equation of the tangent to the conic $4x^2+3xy+2y^2-3x+5y+7=0$ at the point $(1, -2)$. 5

(b) If by transformation from one set of rectangular axes to another with the same origin the expression $ax+by$ changes to $a'x'+b'y'$, then prove that 5

$$a^2+b^2=a'^2+b'^2$$

(c) Find the vertex and the length of the latus rectum of the parabola 5

$$(3x+4y-17)^2=35(4x-3y-6)$$

UNIT—II

3. (a) Show that the tangent to an ellipse at either extremity of a diameter is parallel to the system of chords bisected by the diameter. 5

(Continued)

(b) Prove that if the straight line $\lambda x+\mu y+\nu=0$ touches the parabola $y^2-4px+4pq=0$, then

$$\lambda^2 q+\lambda\nu-p\mu^2=0$$ 5

(c) Find the locus of the point whose distance from the origin is equal to its perpendicular distance from the plane $2x+3y-6z=9$. 5

4. (a) Find the equation of the plane passing through the middle point of the join of the points $(2, -3, 1)$ and $(4, 5, -3)$, and is perpendicular to the line joining the points. 5

(b) If p and p' are the lengths of the two segments of any focal chord of the parabola $y^2=4x$, then show that $p+p'=pp'$. 5

(c) Show that the normal to the rectangular hyperbola $xy=c^2$ at the point t meets the curve again at the point t' such that $t^3t'=-1$. 5

(Turn Over)

UNIT—III

5. *(a)* Prove that the lines

$$\frac{x-2}{4}=\frac{y+1}{3}=\frac{z-3}{5}$$

and

$$x+2y+3z-9=0=2x-y+2z-11$$

are coplanar. 5

(b) Find the equation of the cylinder whose generators are parallel to the line

$$\frac{x}{2}=\frac{y}{5}=\frac{z}{3}$$

and which passes through the origin $z=0,\ 3x^2+4y^2=12$. 5

(c) *(i)* Find the centre and radius of the sphere given by

$$x^2+y^2+z^2+3x-4y+5z+5=0$$

(ii) Find the equation of the tangent plane to the sphere

$$x^2+y^2+z^2=14$$

at the point $(1,\ -2,\ 3)$. 2+3=5

6. *(a)* Find the SD between the y-axis and the line

$$\frac{x-1}{5}=\frac{y-7}{-4}=\frac{z+3}{12}$$

5

(Continued)

(b) Obtain the equation of the sphere having circle

$$x^2+y^2+z^2+10y-4z-8=0,$$
$$x+y+z=3$$

as the great circle. 5

(c) Find the equation of the cone whose vertex is the point (1, 1, 0) and whose guiding curve is $y=0,\ x^2+z^2=4$. 5

UNIT—IV

7. *(a)* Interpret $\vec{a}\times\vec{b}$ geometrically, where $\vec{a}$ and $\vec{b}$ are non-zero vectors. 5

(b) The position vectors of three points A, B and C are $A(1,\ 1,\ -2)$, $B(0,\ -2,\ 2)$ and $C(1,\ 4,\ -6)$. Find the length of the perpendicular from B to AC. 5

(c) If $\vec{r}=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$ and $\hat{s}=\sin t\,\hat{i}-\cos t\,\hat{j}$, then find $\frac{d}{dt}(\vec{r}\cdot\vec{s})$. 5

8. *(a)* Show that

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a}\,\vec{b}\,\vec{d}]\vec{c}-[\vec{a}\,\vec{b}\,\vec{c}]\vec{d}$$

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(Turn Over)

(b) Show that the points $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+4\hat{j}+4\hat{k}$ and $4(\hat{i}+\hat{j}+\hat{k})$ are coplanar. 5

(c) Show that a necessary and sufficient condition that a proper vector $\vec{u}$ always remains parallel to a fixed line is that

$$\vec{u}\times\frac{d\vec{u}}{dt}=\vec{0}$$ 6

UNIT—V

9. (a) If $r=|\vec{r}|$, then show that

$$\vec{\nabla}f(r)=\frac{f'(r)}{r}\vec{r}$$ 5

(b) Find the velocity and acceleration of a particle which moves along the curve $x=2\sin 3t$, $y=2\cos 3t$ and $z=8t$ at any time t. 5

(c) Find the directional derivative of $f=xy+yz+zx$ in the direction of the vector $\hat{i}+2\hat{j}+2\hat{k}$ at the point $(1, 2, 0)$. 5

10. (a) Show that $\vec{\nabla}\times\vec{\rho}=\vec{0}$, where $\vec{\rho}=m\frac{\vec{r}}{r^3}$, m is constant. 5

(b) Show that $\nabla^2\left(\frac{1}{r}\right)=0$, where $r=|\vec{r}|$ and $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. 5

(c) Find the equations of tangent plane to the surface $x^2+y^2-z=0$ at the point $(2, -1, 5)$. 5

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