2/EH–28 (ii) (Syllabus–2015)

2 0 1 6

(April)

STATISTICS

(Elective/Honours)

SECOND PAPER

(Probability Distribution and Statistical Inference)

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Obtain the probability function of binomial distribution from Bernoulli trials, clearly stating the assumptions. Given $X \sim B(n, p)$, obtain the first four central moments through moment-generating function. 3+3=6

   (b) Let X be a binomial variate with parameters n and p. Show that

   (i) $E\left[\frac{X}{n} - p\right]^2 = \frac{pq}{n};\ q = 1 - p$

   (ii) $\text{cov}\left[\frac{X}{n}, \frac{n-X}{n}\right] = \frac{pq}{n}$ 3+3=6

2. (a) State all the conditions under which the binomial distribution can be approximated to Poisson distribution. Derive Poisson distribution as a limiting case of binomial distribution. 2+4=6

(b) The trinomial distribution of two random variables X and Y is given by

$$f_{X,Y}(x, y) = \frac{n}{x!\,y!\,(n-x-y)!} p^x q^y (1-p-q)^{n-x-y}$$

for $x, y = 0, 1, 2, \cdots, n$ and $x + y \leq n$, where $0 \leq p$, $0 \leq q$ and $p + q \leq 1$.

(i) Find the marginal distributions of X and Y.

(ii) Find the conditional distributions of X and Y.

(iii) Hence obtain $E(Y|X = x)$ and $E(X|Y = y)$. 2+2+2=6

UNIT—II

3. (a) Write down the probability density function of a uniform random variable X and obtain the expression for its rth raw moments. Hence or otherwise obtain its mean and variance. 1+2+3=6

(b) Define exponential distribution. Show that the exponential distribution 'lacks memory', that is, if X has an exponential distribution, then for every constant $a \geq 0$, $P(Y \leq x \,|\, X \geq a) = P(X \leq x)$, $\forall\, x$, where $Y = X - a$. 2+3=5

4. (a) Write down the density function of normal distribution and mention its properties. Show that the odd order central moments of normal distribution vanish. 1+2+3=6

(b) If the first two cumulants are 1 and 2 respectively, then write down the normal probability function. 2

(c) If $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ are two independent normal variates, then show that $X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ and $X_1 - X_2 \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$. 3

UNIT—III

5. (a) What do you mean by sampling distribution of a statistic? Obtain the sampling distribution of sample sum from a Poisson population. 2+4=6

(b) Define chi-square statistic and write the p.d.f. of a chi-square variate with n d.f. Show that the sum of two independent chi-square variates is also a chi-square variate. 2+3=5

6. (a) State and prove Tchebyshev's inequality. Write the p.d.f. of t and F distributions and their properties. 3+3=6

(b) State weak law of large numbers and mention its necessary and sufficient conditions. Write a note on central limit theorem. 3+2=5

UNIT—IV

7. (a) Distinguish between an estimate and an estimator. Explain the maximum likelihood estimator of a parameter and write its properties. 2+2+2=6

(b) Mention the characteristics of a good estimator. Show that

$$\frac{\Sigma x_i (\Sigma x_i - 1)}{n(n-1)}$$

is an unbiased estimator of θ^2, for the sample $x_1, x_2, x_3, \cdots, x_n$ drawn on X which takes the value 0 or 1 with respective probabilities $(1-\theta)$ and θ. 2+3=5

8. (a) Define a consistency estimator. Prove that in sampling from $N(\mu, \sigma^2)$ population, the sample mean is a consistent estimator of μ. Define minimum variance unbiased estimator (MVUE). 1+2+2=5

(b) Discuss the problem of interval estimation. Obtain the $100(1-\alpha)\%$ confidence intervals for the parameters (i) θ and (ii) σ^2, when θ is known and equal to μ of the normal distribution

$$f(x, \theta, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2\right)}, \quad -\infty < x < \infty$$

2+2+2=6

UNIT—V

9. (a) Distinguish with example between null and alternative hypotheses. Write a note on different types of errors. Define the terms (i) level of significance, (ii) power of the test and (iii) p-values. 1+2+3=6

(b) Obtain the test statistic for testing the significance of an observed regression. 2

(c) Write a note on t-test for single mean and state the assumptions involved in the test. 2+1=3

10. *(a)* What are large sample tests? Give an example. If X be the number of successes in n trials with constant proportion P of success, then derive the test statistic for the test of significance of population proportion of success. 2+4=6

(b) What is a contingency table? For a 2×2 contingency table

a	b
c	d

prove that the chi-square test of independence gives

$$\chi^2 = \frac{N(ad - bc)^2}{(a+c)(b+d)(a+b)(c+d)}$$ 2+3=5

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