2nd Copy

3/EH–29 (iii) (Syllabus–2015)

2016

(October)

MATHEMATICS

(Elective/Honours)

(Algebra—II and Calculus—II)

(GHS–31)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. (a) Prove that the set $\{1, -1, i, -i\}$ is a finite Abelian group of order 4 with respect to multiplication. $[i^2 = -1]$ 3

   (b) Show that every subgroup of a cyclic group is cyclic. 4

(c) Answer the following with justification : 2×3=6

(i) Can an Abelian group have a non-Abelian subgroup?

(ii) Can a non-Abelian group have an Abelian subgroup?

(iii) Can a non-Abelian group have a non-Abelian subgroup?

(d) Prove that intersection of any two subgroups of a group is a subgroup. 2

2. (a) If G is a finite group, show that for each $a \in G$, there exists a positive integer n such that $a^n = e$, where e is the identity element of a group G. 5

(b) Show that any two left cosets of a subgroup H in a group G have the same (finite or infinite) number of elements. 5

(c) Show that an infinite cyclic group has exactly two generators. 5

UNIT—II

3. (a) Solve $x^4 - x^3 + 3x^2 + 31x + 26 = 0$, if one of the roots of the given equation is $2-3i$. 5

(b) Find the polynomial $f(x+2)$, when

$$f(x) = x^4 - 3x^3 + 4x^2 - 2x + 1$$ 4

(c) Remove the second term of the equation

$$x^3 + 6x^2 + 12x - 19 = 0$$

and then solve the given equation. 6

4. (a) (i) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, form the equation whose roots are

$$\beta\gamma + \frac{1}{\alpha}, \quad \gamma\alpha + \frac{1}{\beta}, \quad \alpha\beta + \frac{1}{\gamma}$$ 4

(ii) If $z = \cos 2\theta + i \sin 2\theta$
$w = \cos 2\phi + i \sin 2\phi$

show that

$$z^m w^n + \frac{1}{z^m w^n} = 2\cos 2(m\theta + n\phi)$$ 2

(b) If the equation

$$3x^4 + 4x^3 - 60x^2 + 96x - k = 0$$

has four real and unequal roots, show that k must lie between 32 and 43. 4

(c) Solve the equation $x^3 - 18x - 35 = 0$ by Cardan's method. 5

UNIT—III

5. (a) Prove that the convergent sequence is bounded. Is the converse true? Justify with an example. 3+1=4

(b) Show that if $x_n = \dfrac{3n+1}{n+2}$, then the sequence $\{x_n\}$ is strictly increasing. Is the sequence convergent? Justify your answer. Also find its limit. 3+2+1=6

(c) Define Cauchy sequence. Is the sequence $\{n^2\}$ a Cauchy sequence? Justify your answer. 2+3=5

6. (a) Test the convergence of the following series (any *two*) : 3×2=6

(i) $\sum\limits_{n=1}^{\infty} \dfrac{n!}{n^n}$

(ii) $\sum\limits_{n=1}^{\infty} \dfrac{2n+1}{(n+1)^2}$

(iii) $\sum\limits_{n=1}^{\infty} \left(\dfrac{1}{\sqrt{n}} + \dfrac{1}{\sqrt{n-1}}\right)$

(b) State Leibnitz's theorem for alternating series. Show that $1 - \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} - \dfrac{1}{\sqrt{4}} + \cdots$ converges. 2+3=5

(c) Define a power series. Find the interval of convergence of $\sum\limits_{n=0}^{\infty} \dfrac{x^n}{n!}$. 1+3=4

UNIT—IV

7. (a) State and prove Cauchy's mean value theorem. 1+4=5

(b) Find the asymptotes of the curve

$$x^3 - 2y^3 + xy(2x - y) + y(x - y) + 1 = 0$$ 5

(c) (i) Find the approximate value of log 10·1 by the use of differentials. Given that $\log_{10} e = 0{\cdot}4343$. 2

(ii) Show that of all rectangles of a given area, the square has the smallest perimeter. 3

8. (a) Let the function be defined by

$$f(x, y) = \begin{cases} \dfrac{2xy}{\sqrt{x^2+y^2}}, & \text{when } x^2+y^2 \neq 0 \\ 0, & \text{when } x = 0 = y \end{cases}$$

Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. 6

(b) State and prove Euler's theorem on homogeneous function of three variables x, y, z. Applying Euler's theorem to the function $V = \tan^{-1} \dfrac{x^3+y^3}{x-y}$, show that $x\dfrac{\partial V}{\partial x} + y\dfrac{\partial V}{\partial y} = \sin 2V$. 1+4+4=9

UNIT—V

9. *(a)* Expand $\log(1+x)$ in a finite series in powers of x with Cauchy's form of remainder. 4

(b) State and prove Taylor's theorem in infinite form with Lagrange's form of remainder. 6

(c) State and prove the fundamental theorem of integral calculus. 1+4=5

10. *(a)* Find the volume of the solid of revolution obtained by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the minor axis. 4

(b) Find the length of arc of the given curves $x = e^\theta \sin\theta$, $y = e^\theta \cos\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$. 5

(c) Evaluate $\int_2^4 \int_{4/x}^{\frac{20-4x}{8-x}} (4-y)\,dy\,dx$ by changing the order of integration. 6

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