2017

(October)

MATHEMATICS

(Elective/Honours)

(GHS–11)

(Algebra—I and Calculus—I)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* If $f(x+1)=x^2-3x+2$, then show that $f(x)=x^2-5x+6$. 4

   (b) Let A, B, C are non-empty sets such that $A\times B=A\times C$. Show that $B=C$. 4

   (c) If $f(x)=\frac{1}{1-x}$, then find the value of $f[f\{f(x)\}]$. 3

(Turn Over)

(d) Find the domain and the range of the function $f(x) = \frac{|x|}{x}$. Also draw the graph of $f(x)$. 2+2=4

2. (a) In an examination, 80 students secured first-class marks in Mathematics or English. Out of these, 50 students secured first-class marks in Mathematics only, and 10 students in English and Mathematics both. How many students secured first class in English only? 4

(b) Let $\mathbb{Z}$ be the set of all integers and a relation R is defined as

$$R = \{(a, b) \mid a - b \text{ is even}\}$$

Is it an equivalence relation? Justify. 4

(c) Show that the limit

$$\lim_{x \to 2} \frac{|x-2|}{x-2}$$

does not exist. 4

(d) Evaluate : 3

$$\lim_{x \to \infty} \frac{x^2 + 3x + 2}{x^3 + x - 4}$$

(Continued)

UNIT—II

3. (a) A mapping $f : \mathbb{N} \to \mathbb{N}$ is defined as $f(x) = x^3$. Show that f is one-one but not onto. 2

(b) Show that the matrix

$$A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$$

is nilpotent. Find its index. 3

(c) A and B are two non-singular matrices. Prove that $(AB)^{-1} = B^{-1}A^{-1}$. 4

(d) Find, applying elementary operations, the rank of the matrix.

$$\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$ 6

4. (a) Solve the system of equations, if consistent : 7

$$2x - y + 3z - 9 = 0$$
$$x + y + z - 6 = 0$$
$$x - y + z - 2 = 0$$

(Turn Over)

(b) Let A and B be any square matrices of same order. Show that $A + A^{\theta}$ is Hermitian and $A - A^{\theta}$ is skew-Hermitian. 4

(c) A and B are two symmetric matrices. Show that AB is symmetric if and only if A and B commute. 4

UNIT—III

5. (a) Let $f(x)$ be a continuous function in a closed interval and does not take the value 0 there. Prove that $f(x)$ keeps the same sign throughout the interval. 4

(b) Find the equation to the tangent to the curve $y = x^2 + 4x - 16$ which is parallel to the line $3x - y + 1 = 0$. 3

(c) Find the derivative of $x \log x$ from the first principle. 4

(d) If

$$y = \tan^{-1}\left(\frac{t}{\sqrt{1-t^2}}\right) \text{ and } x = \sec^{-1}\left(\frac{1}{2t^2 - 1}\right)$$

then show that $\frac{dy}{dx}$ is independent of t. 4

6. (a) A 20-feet ladder leans against a building while its base is drawn away from the wall at the rate 2 ft/sec. How fast is the top of the ladder descending when the ladder is inclined at an angle 60° to the horizontal? 3

(b) Evaluate any *two* of the following : 3×2=6

(i) $\lim_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

(ii) $\lim_{x \to 0} x \log(\tan x)$

(iii) $\lim_{x \to 0} (\sin x)^{\tan x}$

(c) Find y_n, if $y = \log x$. 3

(d) If $xy = \sin(x + y)$, then find $\frac{dy}{dx}$. 3

UNIT—IV

7. (a) Evaluate any *two* of the following : 2½×2=5

(i) $\int \sec^3 x \, dx$

(ii) $\int \frac{e^x}{x}(1 + x \log x) \, dx$

(iii) $\int \frac{x^2}{x^2 - 4} \, dx$

(b) Obtain a reduction formula for $\int \sin^m x \cos^n x \, dx$. Using this formula, obtain the value of $\int_0^{\pi/2} \sin^6 x \cos^8 x \, dx$. 4+2=6

(c) Show that

$$\int_0^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)} = \frac{\pi}{2ab(a+b)} \quad a, b > 0$$ 4

8. *(a)* Using the properties of definite integral, show that

$$\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\log 2$$ 5

(b) Find the value of $\int_0^1 x^3 dx$ by the method of summation. 4

(c) Evaluate : 4

$$\lim_{n\to\infty}\left[\frac{1+2^{10}+3^{10}+\cdots+n^{10}}{n^{11}}\right]$$

(d) Show that

$$\int_0^2 |1-x|\,dx = 1$$ 2

UNIT—V

9. *(a)* Show that $y = e^{-x}(A\cos x + B\sin x)$ is the solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$ 3

(b) Prove that for any straight line $\frac{d^2y}{dx^2} = 0$. 2

(c) Solve : 4

$$\frac{dy}{dx} + \frac{y}{x}\log y = \frac{y}{x^2}(\log y)^2$$

(d) Solve any *two* of the following : 3×2=6

(i) $(6x-8y-5)\,dy = (3x-4y-2)\,dx$

(ii) $(e^x+1)y\,dy + (y+1)e^x dx = 0$

(iii) $x^2 dy = (x^2+5xy+4y^2)\,dx$

(iv) $x\,dx + y\,dy + (x^2+y^2)\,dy = 0$

10. *(a)* Solve any *two* of the following : 4×2=8

(p stands for $\frac{dy}{dx}$)

(i) $x - yp = ap^2$

(ii) $p^2 - p(e^x + e^{-x}) + 1 = 0$

(iii) $y = (1+p)x + ap^2$

(b) Find the general and singular solution of $y = px + \sqrt{a^2p^2 + b^2}$. 4

(c) Show that the equation to the curve whose slope at any point is equal to $y + 2x$ and which passes through origin is $y = 2(e^x - x - 1)$. 3

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