5/H–24 (v) (Syllabus–2015)

2019

(October)

PHYSICS

(Honours)

[PHY-05(T)]

(**Mathematical Physics, Quantum Mechanics**)

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Question No. **1** which is compulsory and *any* **four** from the rest

1. *(a)* Define curl of a vector field $\vec{A}$. When is $\vec{A}$ irrotational? 1+1=2

(b) Show that every eigenvalue of a Hermitian operator is real. 4

(c) Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ by the method of seperation of variables if 4

$$u(0, y) = 8e^{-3y}$$

(d) Define covariant and contravariant tensors. 2

(*Turn Over*)

2. *(a)* State and prove Gauss divergence theorem of a vector function. 1+4=5

(b) Show that

$$\int_S \vec{B} \cdot \vec{n}\, dS = 0$$

if $\vec{B} = \vec{\nabla} \times \vec{A}$ for any closed surface S and $\hat{n}$ being the unit normal outward vector to S. 3

(c) Find the eigenvalues of the matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$ 3

3. *(a)* If $f(z) = u(x, y) + iv(x, y)$ is an analytic function in a domain, then obtain the Cauchy-Riemann conditions

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$ 4

(b) State and prove Cauchy's integral formula. Use this formula to evaluate

$$\int_C \frac{z dz}{(9 - z^2)(z + 1)}$$

where C is a circle $|z| = 2$. 3+4=7

4. *(a)* In Legendre's polynomial, use the generating function

$$\left(1 - 2xz + z^2\right)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} z^n P_n(x)$$

to obtain the recurrence relation 4

$$nP_n = (2n - 1)n\, P_{n-1} - (n - 1)P_{n-2}$$

(b) Obtain the condition of orthoganality of Legendre's polynominal. 4

(c) Write down Rodregue's formula for Legendre's polynomial. Use this formula to show that

$$P_2(x) = \frac{1}{2}\left(3x^2 - 1\right)$$ 1+2=3

5. *(a)* Define gamma function and hence show that

(i) $\Gamma n + 1 = n\Gamma n$; *(ii)* $\Gamma\frac{1}{2} = \sqrt{\pi}$ 1+2+2=5

(b) What is beta function? Obtain the relation between gamma function and beta function. 1+5=6

6. *(a)* Derive Heisenberg's uncertainity relation

$$\Delta p_x \Delta x \geq \frac{\hbar}{2}$$ 7

by using operator method.

(b) What is a Hermitian operator? Show that the product of two Hermitian operators is Hermitian only if they commute. 1+3=4

(Turn Over)

7. (a) Determine the energy level and the corresponding normalised eigen-functions of a particle in one-dimensional potential well of the form

$$V(x) = \infty \text{ for } x < 0 \text{ and for } x > a$$
$$= 0 \text{ for } 0 < x < a$$

What are the boundary conditions for the problem? Is the wave function continuous everywhere? 3+3+1+1=8

(b) Show that every tensor of second rank can be resolved into symmetric and anti-symmetric tensors. 3

8. (a) Find the values of $[L_x, P_x]$ and $[L^2, L_y]$. 1½+1½=3

(b) If σ is the Pauli's spin matrix, show that

$$\left[\sigma_x, \sigma_y\right] = 2i\sigma_z$$ 2

(c) Write the Schrödinger equation for hydrogen atom in spherical polar coordinates. Solve the radial part of this equation to obtain the eigenvalues of energy. 1+5=6

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