5/H–28 (v) (Syllabus–2015)

2019

(October)

STATISTICS

(Honours)

[STH-51 (TH)]

(**Mathematical Methods and Distribution Theory**)

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Attempt **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* What do you mean by numerical differentiation? Hence, derive the numerical differentiation formula based on Newton's forward interpolation formula. 8

(b) Write notes on *(i)* solution of algebraic equation by the method of iteration and *(ii)* by Newton-Raphson method. 4

(*Turn Over*)

2. *(a)* Define partial derivatives of a function of two variables.
Given

$$f(x,y) = \frac{xy}{x^2+y^2},$$
$$\text{where } (x,y) \neq (0,0)$$
$$f(0,0) = 0$$

Show that both the partial derivatives exist at $(0,0)$ but the function is not continuous at $(0,0)$. 6

(b) Find all the maxima and minima of the function given by

$$f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$$ 6

UNIT—II

3. *(a)* Define linear independence and dependence of vectors with examples of each.

Show that the vectors $x_1 = (1,2,3)$, $x_2 = (3,-2,1)$ and $x_3 = (1,-6,-5)$ form a linearly independent set. 4+3=7

(b) Let

$$A^{-1} = \begin{bmatrix} \frac{5}{7} & \frac{1}{7} \\ \frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

Then evaluate $A^2 + 2A$. 4

4. *(a)* State and prove Cayley-Hamilton theorem. 1+5=6

(b) Show that the quadratic form

$$2x^2 + 9y^2 + 4z^2 + 8xy + 6yz + 6zx$$

is not positive definite. 5

UNIT—III

5. *(a)* Define conditional expectation of a continuous function $g(X,Y)$, given that $Y = y_j$.

Show that the variance of X can be regarded as consisting of two parts, the expectation of the conditional variance and the variance of the conditional expectation. Symbolically

$$V(X) = E\left(V\left(\frac{X}{Y}\right)\right) + V\left[E\left(\frac{X}{Y}\right)\right]$$ 1+3=4

(b) Two random variables X and Y have the following joint density function :

$$f(x,y) = 2 - x - y;\ 0 \le x \le 1,\ 0 \le y \le 1$$
$$= 0, \quad \text{otherwise}$$

Find—

(i) marginal probability function of X;

(ii) conditional density function of X; given $Y = y$;

(iii) variance of X. 4

(c) A box contains a white balls and b black balls. c balls are drawn at random. Find the expected value of the number of white balls drawn. 3

6. (a) Define conditional distribution function and mention different properties. 4

(b) If X and Y are random variables, prove that $E(X+Y)=E(X)+E(Y)$. 2

(c) Let 5

$$f(x, y) = 8xy, \quad 0 < x < y < 1$$
$$f(x, y) = 0 \quad \text{elsewhere}$$

Find :

(i) $E(Y \mid X = x)$

(ii) $E(XY \mid X = x)$

(iii) $\text{var}(Y \mid X = x)$

UNIT—IV

7. (a) Define negative binomial distribution. Give the examples, where negative binomial distributions are applicable. Obtain the mean and variance of this distribution.

Show that negative binomial distribution tends to Poisson distribution under some conditions. 2+2+2=6

(b) If a positive random variable X follows log-normal distribution, then show that $\log_e x$ is normally distributed. 3

(c) Explain how you will use hyper-geometric model to estimate the number of fishes in a lake. 2

8. (a) If X and Y are independent gamma variates with parameters μ and ν respectively, then show that $U = X + Y$ and $Z = \frac{X}{Y}$ are independent and that U is a $\gamma(\mu+\nu)$ variate and Z is a $\beta_2(\mu, \nu)$ variate. 5

(b) A sample of n values is drawn from a population whose probability density function is ae^{-ax}, $x > 0$, $a > 0$. If $\bar{X}$ is mean of the sample, show that $na\bar{X}$ is a $\gamma(n)$ variate and prove that 6

$$E(\bar{X}) = \frac{1}{a} \quad \text{and} \quad SE(\bar{X}) = \frac{1}{a\sqrt{n}}$$

UNIT—V

9. (a) Obtain the sampling distribution of sample mean in random sample from a normal population. 5

(b) Define t-distribution. Mention its applications. 1+2=3

(c) Show that limiting form of t-distribution is standard normal distribution. 3

10. *(a)* Derive the probability density function of chi-square distribution. Obtain the mean and variance of chi-square distribution by using cumulant generating function. 3+2=5

(b) Establish the relation between *F*- and χ^2-distribution. 6

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