1/EH–29 (i) (Syllabus–2015)

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(February)

MATHEMATICS

(Elective/Honours)

(Algebra—I and Calculus—I)

(GHS–11)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking one from each unit

UNIT—I

1. *(a)* Prove that for two non-empty sets A and B, $(A \quad B)^c \quad A^c \quad B^c$, where A^c and B^c are the complements of A and B respectively. 3

(b) If $f(x \quad 1) \quad x^2 \quad 5x \quad 2$, then show that $f(x) \quad x^2 \quad 7x \quad 8$. 4

(c) If $f(x) \quad \frac{1}{1 \quad x}$, find the value of $f[f\{f(x)\}]$. 3

(d) A function f is defined as

$$f(x) \quad \begin{matrix} x^2 \quad 2x \quad a, & x \quad 0 \\ 3 \qquad , & x \quad 0 \end{matrix}$$

Find the value of a for which the function f is continuous at $x \quad 0$. 3

(e) Find the domain of the function

$$f(x) \quad \frac{x^2 \quad 4}{x \quad 2}$$ 2

2. *(a)* Let S be the set of all straight lines on a plane. A relation R is defined on S as lRm if and only if l is perpendicular to m, $\quad l, m \quad S$. Examine if R is *(i)* reflexive, *(ii)* symmetric and *(iii)* transitive. Is R an equivalence relation on S? Justify. 1½+1½+1½+½=5

(b) Use - definition to show that

$$\underset{x \quad 3}{\text{Lt}} \frac{3x^2 \quad 27}{x \quad 3} \quad 18$$ 4

(c) Let $f, \; g : \mathbb{R} \quad \mathbb{R}$ be functions defined as $f(x) \quad \cos x$ and $g(x) \quad e^x$. Obtain $f \circ g$ and $g \circ f$. Is $f \circ g \quad g \circ f$? 2+2+½=4½

(d) A and B are two sets as given below :

$$A \quad \{p, q, r\}\ ;\ B \quad \{a, b\}$$

Obtain $A \quad B$ and state the value of $n(B \quad A)$. 1+½=1½

UNIT—II

3. (a) Give examples of functions $f : A \quad B$ such that f is

(i) one-one but not onto

(ii) onto but not one-one

(iii) Both one-one and onto

(iv) Neither one-one nor onto 1+1+1+1=4

(b) Prove that every square matrix is uniquely expressible as the sum of a symmetric and a skew-symmetric matrices. 4

(c) Show that the system of equations

$$\begin{array}{cccc} x & y & z & 6 \\ x & 2y & 3z & 14 \\ x & 4y & 7z & 30 \end{array}$$

is consistent and solve them. 7

4. (a) If A is a non-singular matrix, show that $|\text{adj } A| \quad |A|$. 4

(b) Reduce the following matrix to its normal form and hence obtain its rank

$$\begin{array}{cccc} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{array}$$

6+1=7

(c) If A, B, C are $2 \quad 2$ matrices such that $AB \quad AC$, does it imply that $B \quad C$? Justify your answer. 4

UNIT—III

5. (a) A toy spherical balloon being inflated, the radius is increasing at the rate of $\frac{1}{11}$ cm per second. At what rate would the volume be increasing at the instant, when the radius, $r \quad 7$ cm? 4

(b) Find $\frac{dy}{dx}$ of the following (any *one*) : 4

(i) $y^x \quad x^y$

(ii) $y \quad \tan^{\ 1} \dfrac{1 \quad \cos x}{1 \quad \sin x}$

(c) Show that the equation $x^4 \quad x^3 \quad 3 \quad 0$ has a real root between 1 and 2. Also state the theorem that you use. 1+2=3

(d) Evaluate by L'Hospital's rule : 2+2=4

(i) $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$

(ii) $\lim\limits_{x \to \infty} \dfrac{x^3}{e^x}$

6. (a) Let $y = \tan^{-1} x$, show that

(i) $(1 + x^2)y_1 = 1$

(ii) $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$

2+4=6

(b) When is a function said to be uniformly continuous in an interval? Show that the function $f(x) = x^2$ is uniformly continuous in $[-1, 1]$. 1+3=4

(c) If $y = \sin^{-1} x$, prove that

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0 \quad 5$$

UNIT—IV

7. (a) Evaluate (any *two*) : 3×2=6

(i) $\displaystyle\int \frac{1}{x(x+1)^2}\, dx$

(ii) $\displaystyle\int \frac{dx}{1 + \tan x}$

(iii) $\displaystyle\int \frac{x^2 dx}{x^2 + 4}$

(b) Show that

$$\int \tan^5 x\, dx = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| \quad 4$$

(c) Using the properties of definite integral, show that

$$\int_0^1 \frac{\log(1+x)}{1+x^2}\, dx = \frac{\pi}{8}\log 2 \quad 5$$

8. (a) Prove that

$$\lim_{n \to \infty}\left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \cdots + \frac{1}{8n}\right] = \frac{3}{8} \quad 4$$

(b) Evaluate by the method of summation

$$\int_1^2 (x^2 + 2)\, dx \quad 5$$

(c) Evaluate the following integral if convergent : 3

$$\int_0^\infty \frac{dx}{x^2 + 2x + 2}$$

(d) If $I_n = \int_0^{\pi/2} \sin^n x\, dx$, where n is a positive integer, $n > 1$; prove that

$$I_n = \frac{n-1}{n} I_{n-2} \quad 3$$

UNIT—V

9. *(a)* Show that $y \quad e^{\;x}(A\cos x \quad B\sin x)$ is the solution of the differential equation

$$\frac{d^2y}{dx^2} \quad 2\frac{dy}{dx} \quad 2y \quad 0$$ 3

(b) Solve any *three* of the following : 3×3=9

(i) $(6x \quad 8y \quad 5)\,dy \quad (3x \quad 4y \quad 2)\,dx$

(ii) $(x^2 \quad yx^2)dy \quad (y^2 \quad xy^2)\,dx \quad 0$

(iii) $x\,dx \quad y\,dy \quad (x^2 \quad y^2)\,dy \quad 0$

(iv) $x\frac{dy}{dx} \quad y \quad \cos^{\;1}\frac{1}{x}$

(c) Solve the following : 3

$$\frac{dy}{dx} \quad \frac{y}{x} \quad y^2$$

10. *(a)* Solve any *two* of the following : 3½×2=7

(i) $p^2 \quad p(x \quad y) \quad xy \quad 0$

(ii) $y \quad (1 \quad p)x \quad p^2$

(iii) $y \quad yp^2 \quad 2px$

(b) Find the general and singular solution of $y \quad px \quad \sqrt{a^2p^2 \quad b^2}$. 4

(c) Find the equation of the orthogonal trajectories of the family of curves

$$\frac{x^2}{a^2} \quad \frac{y^2}{a^2 \quad ^2} \quad 1$$

where is a parameter. 4

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