1/EH–29 (i) (Syllabus–2019)

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(February)

MATHEMATICS

(Elective/Honours)

(Algebra—I and Calculus—I)

(GHS–11)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking one from each unit

UNIT—I

1. (a) Find the domain of the function

$$f(x) = \frac{1}{\sqrt{|x| - x}}$$ 3

(b) If $f(x) = \frac{x-1}{x+1}$, show that

$$\frac{f(x) - f(y)}{1 + f(x)f(y)} = \frac{x - y}{1 + xy}$$ 3

(c) Draw the graph of the function $f(x) = [x]$, where $[x]$ denotes the greatest integer not greater than x. 3

(d) Let $f(x) = |x|$, show that

$$\operatorname*{Lt}_{h \to 0} \frac{f(h) - f(0)}{h}$$

does not exist. 3

(e) Examine the continuity of the function f given as in Q. No. **1**(*d*) on $\mathbb{R}$. 3

2. (a) Which of the following statements are true? 2

For a set A

(i) $A \in P(A)$

(ii) $A \subseteq P(A)$

(iii) $\{A\} \in P(A)$

(iv) $\{A\} \subseteq P(A)$

(b) Prove that for any two sets A and B $A - B = A \cap B^c$. 2

(c) Show that

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = 1$$ 3

(d) If R is a relation on $\mathbb{Z} - \{0\}$ and xRy if and only if $xy > 0$, prove that R is an equivalence relation. 3

(e) If A and B are two sets and $A \quad A \quad B \quad B$, then prove that $A \quad B$. 2

(f) $f : Q \quad Q$ is defined by $f(x) \quad 3x \quad 4$. Show that f is invertible and find $f^{\quad 1}$. 3

UNIT—II

3. *(a)* Define, with example, the following terms : 2×3=6

(i) Symmetric matrix

(ii) Skew-symmetric matrix

(iii) Diagonal matrix

(b) Reduce the matrix A to the normal form and find its rank, where

$$A \quad \begin{matrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 4 & 0 \\ 2 & 2 & 8 & 0 \end{matrix}$$

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(c) Let A and B are Hermitian matrices, show that $AB \quad BA$ is Hermitian. 3

4. *(a)* Examine if the following system of equations are consistent :

$$\begin{matrix} x & y & 2z & 4 \\ 3x & y & 4z & 6 \\ x & y & z & 1 \end{matrix}$$

If consistent, solve the system. 3+4=7

(b) Using elementary operation find the inverse of the matrix A, where

$$A \quad \begin{matrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{matrix}$$

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(c) Show that the matrix

$$A \quad \begin{matrix} 2 & 2 & 4 \\ 1 & 3 & 4 \\ 1 & 2 & 3 \end{matrix}$$

is idempotent. 2

UNIT—III

5. *(a)* State the intermediate value theorem. Prove that the equation $x^4 \quad x^3 \quad 3 \quad 0$ has a real root between 1 and 2. 2+1=3

(b) Evaluate $\dfrac{dy}{dx}$ of any *two* of the following : 3×2=6

where

(i) $x^y y^x \quad 1$

(ii) $x \quad \sqrt{1 \quad u},\ y \quad \sqrt{1 \quad u}$

(iii) $y \quad \tan^{\quad 1}\sqrt{\dfrac{1 \quad \cos}{1 \quad \cos}}$

(c) Find the points on the curve

$$y \quad 2x^3 \quad 15x^2 \quad 34x \quad 20$$

where the tangents are parallel to $y \quad 2x \quad 0$. 3

(d) From the first principle obtain the derivative of $\frac{1}{\sqrt{x}}$. 3

6. *(a)* If the area of a circle increases at a uniform rate, prove that the rate of increase of the perimeter varies inversely as the radius. 3

(b) Evaluate : 2½×2=5

(i) $\mathop{\mathrm{Lt}}_{x \quad 0} \frac{1}{x^2} \quad \frac{1}{\sin^2 x}$

(ii) $\mathop{\mathrm{Lt}}_{x \quad 0} \frac{e^x \quad \sin x \quad 1}{\log(1 \quad x)}$

(c) If $y \quad \tan^{\quad 1} x$, show that

(i) $(1 \quad x^2)y_1 \quad 1$

(ii) $(1 \quad x^2)y_{n \quad 1} \quad 2nxy_n \quad n(n \quad 1)y_{n \quad 1} \quad 0$

Find also $(y_n)_0$. 1½+3½+2=7

UNIT—IV

7. *(a)* Evaluate any *two* of the following : 3×2=6

(i) $\frac{d}{5 \quad 4\cos}$

(ii) $\frac{dx}{x(x \quad 1)^2}$

(iii) $\sqrt{2ax \quad x^2}\,dx$

(b) Show that

$${}_0^{\;2}\sin^6 \quad \cos^3 \quad d \quad \frac{2}{63}$$ 3

(c) Find by method of summation

$${}_0^1 x^3\,dx$$ 3

(d) Evaluate, if possible,

$${}_0 \frac{dx}{1 \quad \cos x}$$ 3

8. *(a)* Show that

$${}_0 x\log\sin x\,dx \quad \frac{{}^2}{2}\log\frac{1}{2}$$ 4

(b) Evaluate : 4

$$\mathop{\mathrm{Lt}}_{n} \frac{1^2}{n^3 \quad 1^3} \quad \frac{2^2}{n^3 \quad 2^3} \quad \cdots \quad \frac{n^2}{2n^3}$$

(c) If $u_n \quad {}_0^{2} x^n \sin x\, dx,\ n \quad 0$, prove that

$$u_n \quad n(n \quad 1)u_{n \quad 2} \quad n \frac{}{2}^{n \quad 1}$$ 5

(d) Evaluate : 2

$$ {}_0^1 x^3 \sqrt{1 \quad 3x^4}\, dx $$

UNIT—V

9. (a) All the circles which touch the Y-axis at the origin is given by the equation $x^2 \quad y^2 \quad 2cx$. Obtain the differential equation of the family. 3

(b) Solve any *four* of the following : 3×4=12

(i) $(3y \quad 2x \quad 4)\,dx \quad (4x \quad 6y \quad 5)\,dy$

(ii) $x^2 dy \quad (xy \quad 2y^2)dx \quad 0$

(iii) $3e^x \tan y\, dx \quad (1 \quad e^x)\sec^2 y\, dy \quad 0$

(iv) $x\,dy \quad y\,dx \quad \cos\frac{1}{x}\,dx$

(v) $x(x^2 \quad y^2 \quad 4)\,dx \quad y(x^2 \quad y^2 \quad 9)\,dy \quad 0$

(vi) $(1 \quad y^2)\,dx \quad (\tan^{\quad 1} y \quad x)\,dy$

10. (a) Solve any *two* of the following : 2½×2=5

(i) $p(p^2 \quad xy) \quad p^2(x \quad y)$

(ii) $y \quad 2px \quad p^2$

(iii) $p^2 \quad py \quad x \quad 0$

(p stands for $\frac{dy}{dx}$

(b) Find the orthogonal trajectories of the family of confocal conics

$$\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$$

is a parameter. 4

(c) Reduce the equation $(px \quad y)(x \quad py) \quad 2p$ to Clairaut's form by the substitution $x^2 \quad u,\ y^2 \quad v$. 2

(d) Solve any *two* of the following : 2×2=4

(i) $\frac{d^2y}{dx^2} \quad 8\frac{dy}{dx} \quad 25y \quad 0$

(ii) $\frac{d^2y}{dx^2} \quad 2\frac{dy}{dx} \quad y \quad 0$

(iii) $\frac{d^2y}{dx^2} \quad 6\frac{dy}{dx} \quad 25y \quad 0$

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