3/EH–29 (iii) (Syllabus–2019)

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(February)

MATHEMATICS

(Elective/Honours)

(Statics and Calculus—II)

(GHS–31)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. (a) State Lami's theorem. Forces P, Q, R acting along $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, where O is the circum-centre of the triangle ABC, are in equilibrium, show that

$$\frac{P}{a^2(b^2 \quad c^2 \quad a^2)} \qquad \frac{Q}{b^2(c^2 \quad a^2 \quad b^2)} \qquad \frac{R}{c^2(a^2 \quad b^2 \quad c^2)}$$

where a, b, c are the lengths of the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ respectively. 1+4=5

(b) The lines of action of two forces $P \quad Q$, $P \quad Q$ make an angle 2 with one another, and their resultant makes an angle with the bisector of the angle between them. Show that

$$P\tan \qquad Q\tan$$ 5

(c) Prove that a force and a couple in the same plane are equivalent to a single force, equal and parallel to the given single force. 5

2. (a) If the two like parallel forces P and Q acting on a rigid body at A and B be interchanged in position, then show that the point of application of the resultant will be displaced along $\overline{AB}$ through a distance d, where

$$d \quad \frac{P \quad Q}{P \quad Q} AB, \ (P \quad Q)$$ 5

(b) Three forces P, Q, R acting at the vertices A, B, C respectively of a triangle, each perpendicular to the opposite side, keeping it in equilibrium. Prove that

$$P : Q : R \quad a : b : c$$

where a, b, c are the sides of the triangle opposite to A, B, C respectively. 5

(c) Forces of magnitudes P, $2P$, P, $2P$ act along the sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ of the square $ABCD$, and a force of magnitude $P\sqrt{2}$ acts along each of $\overline{BD}$ and $\overline{CA}$. Show that the forces reduce to a couple of moment $2aP$, where a is the side of the square. 5

UNIT—II

3. *(a)* ABC is an equilateral triangle, forces of 4 kg, 2 kg and 1 kg (force) act along the sides $\overline{AB}$, $\overline{AC}$, $\overline{BC}$ respectively, in the senses indicated by the order of the letters. Find the magnitude, direction and line of action of the resultant. 5

(b) A heavy uniform rod is in equilibrium with one end resting against a smooth vertical wall, and the other against a smooth plane inclined to the wall at an angle . Prove that if be the inclination of the rod to the horizon, then

$$\tan \qquad \frac{1}{2}\tan$$ 5

(c) The least force which will move a weight up an inclined plane is P. Show that the magnitude of the least force, acting parallel to the plane, which will move the weight upwards is $P\sqrt{1 \quad ^2}$, being the coefficient of friction of the plane. 5

4. *(a)* Find the centre of gravity of a uniform trapezium lamina. 5

(b) The moments of a system of coplanar forces acting in the (x, y)-plane about $(0, 0)$, $(a, 0)$, $(0, a)$ are aW, $2aW$, $3aW$ respectively. Find the magnitudes of the components parallel to the coordinate axes and the line of action of the single force to which the system is equivalent. 5

(c) A uniform ladder rests in limiting equilibrium with the lower end on a rough horizontal plane and its upper end against a smooth vertical wall. If be the inclination of the ladder to the vertical, then prove that tan 2 , where is the coefficient of friction. 5

UNIT—III

5. *(a)* Prove that a convergent sequence is bounded. Is the converse true? Justify with an example. 4+1=5

(b) Prove that the sequence

$$\frac{4n \quad 3}{n \quad 2}$$

is bounded and monotone increasing. Is the sequence convergent? Justify your answer. Also find its limit. 2+2+1=5 *(c)* State Cauchy's general principle of convergence as applied to sequence and use it to prove that the sequence $\{x_n\}$ is a convergent sequence, when

$$x_n \quad 1 \quad \frac{1}{2!} \quad \frac{1}{3!} \quad \cdots \quad \frac{1}{n!}$$ 2+3=5

6. *(a)* Test the convergence of the following series (any *two*) : 3×2=6

(i) $$_{n \quad 1} \frac{1}{(2n \quad 1)(2n \quad 1)}$$

(ii) $$\frac{4}{1!} \quad \frac{4^2}{2!} \quad \frac{4^3}{3!} \quad \cdots \quad \frac{4^n}{n!} \quad \cdots$$

(iii) $$_{n \quad 1} \quad \frac{n \quad 1}{n}^{n \quad 1} \quad \frac{n \quad 1}{n}^{n}$$

(b) What is an alternating series? State Leibnitz's test for the convergence of an alternating series and hence show that the series

$$1 \quad \frac{1}{2^2} \quad \frac{1}{3^2} \quad \frac{1}{4^2} \quad \cdots$$

converges. 1+1+2=4

(c) Find the interval of convergence of the power series

$$_{n \quad 1} \frac{1 \quad 3 \quad 5 \quad \cdots (2n \quad 1)}{2 \quad 4 \quad 6 \cdots 2n} \frac{x^{2n \quad 1}}{(2n \quad 1)}$$ 5

UNIT—IV

7. *(a)* State and prove Rolle's theorem. 1+4=5

(b) Find the asymptotes of

$$2x^3 \quad 2x^2y \quad 7xy^2 \quad 2y^3 \quad 14xy \quad 7y^2 \quad 4x \quad 5y \quad 0$$ 5

(c) Find the points of inflexion, if any, of the curve $x \quad (\log y)^3$. 5

8. *(a)* Show that for the function $f(x, y)$ defined by $f(x, y) \quad \frac{x \quad y}{x \quad y}$, $x \quad y \quad 0$. $\lim_{x \quad 0} \lim_{y \quad 0} f(x, y)$ and $\lim_{y \quad 0} \lim_{x \quad 0} f(x, y)$ both exist but are unequal. Also show that

$$\lim_{(x, y) \quad (0, 0)} f(x, y)$$

does not exist. 1+1+3=5

(b) State and prove Euler's theorem for a homogeneous function of three variables x, y, z. 1+4=5

(c) If $u \quad x \sin^{\quad 1}(y / x) \quad y \tan^{\quad 1}(x / y)$, then show that $x\frac{u}{x} \quad y\frac{u}{y}$ at $(1, 1)$ is $\frac{3}{4}$. 5

UNIT—V

9. *(a)* Expand $f(x) = \sin x$ in a finite series in powers of x with Lagrange's form of remainder. 4

(b) State and prove the fundamental theorem of integral calculus. 1+5=6

(c) Show that the area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$ square units. 5

10. *(a)* Find the length of the curve $ay^2 = x^3$ between the points $x = 0$ and $x = 5a$. 5

(b) Find the volume of the solid of revolution obtained by revolving the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ about its minor axis. 5

(c) Evaluate

$$\iint_R xy(x^2 + y^2)\, dx\, dy$$

over $R[0, a; 0, b]$. 5

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