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(February)

STATISTICS

(Elective/Honours)

(Categorical Data, Survey Sampling and Design of Experiments)

[STEH–3 (TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* What do you mean by independence of attributes? Give criteria of independence for two attributes. 3

(b) What do you mean by coefficient of contingency? Describe how to use the same for a 2×2 contingency table. 5

(c) If $(AB)\quad(AB)_0$, then with usual notations, prove that

(i) $[(A)\quad(\;)][(B)\quad(\;)]\quad 2N$

$$(AB)^2\quad(\;)^2\quad(A\;)^2\quad(\;B)^2$$

(ii) $\dfrac{(B)(\;)}{N}\quad\dfrac{(AB)}{(B)}\quad\dfrac{(A\;)}{}$ 4

2. *(a)* When are two attributes said to be—

(i) positively associated;

(ii) negatively associated?

Also define complete association and dissociation of the two attributes. 4

(b) Show that if

$$\frac{(A)}{N}\quad x,\ \frac{(B)}{N}\quad 2x,\ \frac{(C)}{N}\quad 3x \text{ and}$$

$$\frac{(AB)}{N}\quad\frac{(BC)}{N}\quad\frac{(CA)}{N}\quad y$$

then the value of neither x nor y can exceed $\frac{1}{4}$. 4

(Notations have their usual meanings.)

(c) In a series of houses actually invaded by smallpox, 70% of the inhabitants are attacked and 85% have been vaccinated. What is the lowest percentage of the vaccinated that must have been attacked? 4

Unit—II

3. *(a)* Distinguish between complete enumeration and sample survey. 3

(b) How does sampling with replacement differ from that of without replacement? Which of them provides a more efficient estimator of the population mean? 2+1=3

(c) Show that, in SRSWOR, the sample mean square is an unbiased estimate of the population mean square, i.e., $E(s^2) \quad S^2$. 5

4. *(a)* Describe the method of determining the sample size in case of simple random sampling with given margin of error and confidence coefficient $(1 \quad)$. 3

(b) Obtain the sampling variance of the mean based on systematic sample and compare the variance with that based on simple random sampling. 5

(c) In simple random sampling with replacement, show that

$$v(\bar{x}) \quad \frac{\quad^2}{n}$$

3

Unit—III

5. *(a)* What is stratified random sampling? When will you use stratified random sampling? Describe the advantages of stratified random sampling. 6

(b) Prove that the variance of the estimate of population mean in stratified random sampling with proportional allocation is less than or equal to variance of the estimate of population mean based on unstratified simple random sampling. 5

6. *(a)* With a cost function $C \quad a \quad c_h n_h$, prove that the variance of the estimated mean $\bar{y}_{st}$ is minimum when n_h is proportional to

$$\frac{N_h S_h}{\sqrt{C_h}}$$

5

(Notations have their usual meanings.)

(b) Obtain the variance of the estimates of the population mean under stratified random sampling with proportional and optimum allocations, respectively and show that under certain conditions to be stated

$$\text{var(opt)} \quad \text{var(prop)}$$

6

UNIT—IV

7. *(a)* Explain the mean of analysis of variances and give its uses. State the basic assumptions in the analysis of variance. 2+1+1=4

(b) State the mathematical model used in analysis of variance in a two-way classification. Discuss the analysis of variance of a two-way classified data with one observation per cell under fixed effect model. Explain the hypotheses to be used. Also, discuss the advantages of this method over one-way classification, if any. 1+4+1+1=7

8. *(a)* Explain the principles of replication, randomization and local control in experimental designs pointing out the role each one plays in the valid and precise interpretation of the data. 8

(b) Define experimental error. What methods are needed to increase the precision of an experiment? 1+2=3

UNIT—V

9. *(a)* What is meant by RBD? Give the analysis of variance for the design stating clearly the mathematical model and the underlying assumptions. 6

(b) Give an outline of the analysis of a variance of a p p Latin square design involving a single-missing observation. 5

10. *(a)* What is a factorial experiment? Define the terms 'main effects' and 'interaction effects' in relation to a 2^3-experiment. 5

(b) What is a treatment contrast? When are two such contrasts said to be orthogonal? Show that in a 2^3-experiment, the main effects and interaction effects are mutually orthogonal. 1+1+4=6

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