2 0 2 2

(February)

MATHEMATICS

(Honours)

(Advanced Dynamics)

Marks : 45

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Answer **three** questions, choosing **one**
from each Unit

All the symbols have their usual meanings

UNIT—I

1. *(a)* Discuss the motion of a particle on a smooth cycloid whose axis is vertical and vertex lowest. 8

   (b) A particle moves under a central repulsive force $= \dfrac{m}{(\text{distance})^3}$ and is projected from an apse at a distance a with velocity V. Show that the equation

---

to the path is $r \cos p \quad a$ and the angle described in time t is $\dfrac{1}{p} \tan^{1} \dfrac{pVt}{a}$,

where $p^2 \quad \dfrac{a^2 V^2}{a^2 V^2}$. 7

2. *(a)* State Kepler's laws of planetary motion. 3

   (b) A particle describes an ellipse about a centre of force at the focus. Show that at any point of its path, the angular velocity about the other focus varies as the square of the normal at the point. 5

   (c) Establish a criterion for the stability of a circular orbit of radius a with the centre at the centre of force, described under the influence of the force $f(r)$. 7

UNIT—II

3. *(a)* Given the moments and products of inertia of a body about three perpendicular and concurrent axes, find the moment of inertia about any other axis through their meeting point. 7

   (b) Show that a uniform solid cuboid of mass M is equimomental with masses $\dfrac{1}{24} M$ at the midpoints of its edges and $\dfrac{1}{2} M$ at its centre. 8

(3)

4. *(a)* A uniform rod *OA* of length $2a$ free to turn about its end *O* revolves with uniform angular velocity about the vertical *OZ* through *O* and is inclined at a constant angle to *OZ*. Find the value of . 8

(b) A rough uniform board of mass m and length $2a$ rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. Prove that the distance through which the board moves is $\dfrac{2Ma}{(M\ m)}$. 7

UNIT—III

5. *(a)* Deduce the equation of motion of compound pendulum. 5

(b) Find the length of the simple equivalent pendulum of a circular wire with axis tangent to the plane at any point of the arc. 4

(c) A uniform rod of mass m and length $2a$ can turn freely about one end which is fixed. Find its motion if it is started with angular velocity from the position in which it hangs vertically. 6

(4)

6. *(a)* Discuss the motion of a uniform solid circular cylinder down a rough inclined plane. 7

(b) A uniform solid cylinder is placed with its axis horizontal on a plane whose inclination to the horizon is . Show that the least coefficient of friction between it and the plane, so that it may roll and not slide, is $\dfrac{1}{3}\tan$. 8

★ ★ ★