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(February)

MATHEMATICS

(Honours)

(Advanced Calculus—I)

[GHS–52]

Marks : 45

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **three** questions, choosing **one** from each Unit

UNIT—I

1. *(a)* What do you mean by a partition of an interval $[a, b]$? Define the upper sum and the lower sum of a bounded function f with respect to some partition P. 1+1+1=3

(b) Show that the beta function

$$(m, n) \quad {}_0^1 x^{m \;\; 1}(1 \quad x)^{n \;\; 1} dx$$

exists if and only if m and n are both positive. Determine whether the following beta function is convergent or not :

$$ {}_0^1 \frac{x}{(1 \quad x)} dx$$

5+3=8

(c) Evaluate :

$$ {}_0 \frac{\tan^{\;1} ax \quad \tan^{\;1} bx}{x} dx$$

4

2. *(a)* Show that every continuous function is integrable. 5

(b) Show that

$$ {}_0^{\overline{2}} \frac{x^m}{\sin^n x} dx$$

exists if and only if $n \quad m \quad 1$. 5

(c) State Dirichlet's theorem. Hence prove that

$$ {}_0 \frac{\sin x}{x} dx$$

is convergent. 1+4=5

UNIT—II

3. *(a)* If $f(x, y)$ is a continuous function and if f_y is continuous in $[a, b; c, d]$, then show that the integral

$$(y) \quad {}_a^b f(x, y)$$

is differentiable and that

$$(y) \quad {}_a^b f_y(x, y) dx$$

for all $y \quad [c, d]$. 5

(b) If $|a|$ 1, then show that

$$_{0}\log(1 \quad a\cos x)dx \qquad \log\frac{1}{2} \quad \frac{1}{2}\sqrt{1 \quad a^2}$$ 7

(c) Establish the uniform convergence of

$$_{0} e^{\quad x^2} \cos x dx$$ 3

4. *(a)* If a function $f(x, y)$ is continuous in $[a, b; c, d]$, then prove that

$$\overset{d}{\underset{c}{}}\ \overset{b}{\underset{a}{}} f(x, y)dx \ \ dy \qquad \overset{b}{\underset{a}{}}\ \overset{d}{\underset{c}{}} f(x, y)dy \ \ dx$$ 5

(b) If $|a|$ 1, then show that

$$_{0}\frac{\log(1 \quad a\cos x)}{\cos x}dx \qquad \sin^{\ 1} a$$ 6

(c) State Weierstrass M-test and provide an illustration of its application. 2+2=4

UNIT—III

5. *(a)* Verify Green's theorem by evaluating in two ways the line integral

$$(x^3 \quad y^2)dx \quad (x^2 \quad y^3)dy$$

taken along the boundary of the polygon whose vertices are (0, 0), (1, 0), (2, 1) and (0, 1). 9

(b) Evaluate :

$$\frac{\sqrt{a^2b^2 \quad b^2x^2 \quad a^2y^2}}{\sqrt{a^2b^2 \quad b^2x^2 \quad a^2y^2}}dxdy$$

The field of integration being the positive quadrant of the ellipse

$$\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$$ 6

6. *(a)* Evaluate the repeated integral

$$\overset{b}{\underset{0}{}}\ \overset{a}{\underset{0}{}} y e^{xy} dxdy$$ 4

(b) Change the order of integration of the double integral

$$_{0}\ _{x}\frac{e^{\quad y}}{y}dydx$$

and hence find the value. 4

(c) Using the result that

$$(m, n) \qquad \frac{(m) \ (n)}{(m \quad n)}$$

prove that $\frac{1}{2} \qquad \sqrt{\ }$. 7

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