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(February)

STATISTICS

(Honours)

(Mathematical Methods and Distribution Theory)

[STH–51 (TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* What is numerical integration? Using Newton's forward difference formula, derive the general formula for numerical integration. 2+3=5

(b) Explain briefly the methods of false position and iteration using regula falsi for determining the real roots of a numerical equation. Explain the condition under which the iteration method will converge. 2+2+3=7

2. *(a)* Verify that

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

where $f(x, y) = \log\left(\frac{x^2 + y^2}{xy}\right)$. 5

(b) What do you mean by the maximum and minimum of a function $f(x, y)$ of two variables x and y at a point (a, b)? Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 6$. 2+5=7

UNIT—II

3. *(a)* Prove that the linearly independent solutions of the equation $\underset{\sim}{A}\,\underset{\sim}{X} = \underset{\sim}{O}$ is $(n - r)$, where

$$\underset{\sim}{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad \underset{\sim}{X} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ \cdots \\ x_n \end{bmatrix}, \quad \underset{\sim}{O} = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ \cdots \\ 0 \end{bmatrix}$$

and r is the rank of the $m \times n$ matrix $\underset{\sim}{A}$. 9

(b) What are the conditions under which a matrix is in row reduced echelon form? 2

4. *(a)* Define characteristic matrix. Show that every square matrix satisfies its own characteristic equation. 2+4=6

(b) Define the following : 5

(i) Homogeneous system of linear equations

(ii) Trivial solution of homogeneous system of linear equations

(iii) Solution space of homogeneous system of linear equations

(iv) Fundamental set of solutions of homogeneous system of linear equations

(v) Non-homogeneous system of linear equations

UNIT—III

5. *(a)* Define marginal and conditional distribution functions. 2

(b) The joint p.d.f. of two-dimensional random variables X and Y is given by

$$f(x, y) = 4x(1-y);\ 0 < x < 1,\ 0 < y < 1$$

Find the marginal density functions of X and Y and conditional density functions of X and Y. Also find the probability of the event A, where

$$A = \left\{(x, y) : x > \frac{1}{2},\ y > \frac{1}{2}\right\}$$

4+2+3=9

6. *(a)* Define conditional variance. 1

(b) The joint p.d.f. of two-dimensional random variables X and Y is

$$f(x, y) = \frac{1}{8}(6-x-y);\ 0 < x < 2,\ 2 < y < 4$$

Find—

(i) $E(Y|X)$

(ii) $V(Y|X)$

(iii) $V(3X-4Y)$ 6

(c) Let A be an event that could describe a set of values for a random variable and Y be a random variable such that knowing Y gives us useful information about whether or not A has occurred. Compute the probability of A. 4

UNIT—IV

7. *(a)* Write down some experimental results which can be expressed by the hypergeometric probability law. Find the mean and variance of hypergeometric distribution. 2+4=6

(b) How would you derive the probability function of negative binomial distribution from results of Bernoulli experiment? Mention some examples of negative binomial variable. 3+2=5

8. *(a)* What is a gamma variate? Show that if $X \sim N(\mu, \sigma^2)$, then

$$Y = \frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2$$

is a gamma variate. 2+4=6

(b) Define log-normal distribution. Discuss the importance of log-normal distribution. 1+4=5

UNIT—V

9. *(a)* What do you mean by sampling distribution? Obtain the sampling distribution of the sample total for negative binomial distribution. 2+4=6

(b) Define chi-square variate. Show that

$$\sqrt{2\chi^2} \sim N(\sqrt{2n}, 1)$$

where n is the d.f. of χ^2-variate for large n. 1+4=5

10. *(a)* Define F-statistic and derive its distribution. 1+5=6

(b) Show that the t-distribution is symmetric and leptokurtic. 5

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