Odd Semester, 2020

(Held in March, 2021)

MATHEMATICS

(Elective/Honours)

(GHS–11)

(Algebra–I and Calculus–I)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Prove that for any three sets A, B, C, $(A-C)\cap(B-C)=(A\cap B)-C$. 4

(b) Give example of a relation which is—

(i) reflexive but neither symmetric nor transitive;

(ii) reflexive, symmetric but not transitive;

(iii) symmetric and transitive but not reflexive;

(iv) reflexive and anti-symmetric. 1×4=4

(c) If $f: x \to y$ and $g: y \to z$ are one-to-one and onto mappings, then prove that $g \circ f : X \to Z$ is one-to-one and onto and $(g\circ f)^{-1} = f^{-1}\circ g^{-1}$. 5

(d) If $A = \{x, y, z\}$, then find the power set of A. 2

2. *(a)* If $f(x)=\dfrac{1+x}{1-x}$, then prove that $2\cdot f(x)\cdot f(x^2)=1+\{f(x)\}^2$. 4

(b) A survey report reveals that 59% of college students like tea whereas 72% like coffee. Find the possible range of the percentage of college students who like both tea and coffee. 4

(c) Draw the graph of the function

$$f(x)=\begin{cases}3x+2, & x<0\\ x+1, & x\geq 0\end{cases}$$

in the interval [– 2, 2]. Is this function continuous at $x = 0$? 2+2=4

(d) Use ε-δ definition to prove that

$$\lim_{x\to 0} x\sin\frac{1}{x}=0$$ 3

UNIT—II

3. *(a)* Show that the matrix

$$A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$$

is nilpotent and find its index. 2+1=3

(b) Determine if the following system of equations is consistent and if so, find the solution : 4+3=7

$$2x - y + 3z = 8$$
$$-x + 2y + z = 4$$
$$3x + y - 4z = 0$$

(c) Reduce the following matrix to normal form and find its rank : 5

$$A = \begin{bmatrix} 0 & 1 & -3 \\ 2 & 1 & 4 \\ 1 & 2 & 1 \end{bmatrix}$$

4. *(a)* Obtain the inverse of the matrix

$$\begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$$

by using elementary operations. 6

(b) Show that every square matrix A can be uniquely expressed as $P + iQ$ where P and Q are Hermitian matrices. 4

(c) If A and B be two matrices conformable to form the product AB, then show that $(AB)^T = B^T A^T$, where X^T represents the transpose of the matrix X. 5

UNIT—III

5. *(a)* Find $\dfrac{dy}{dx}$ (any *two*) : 4×2=8

(i) $(\cos x)^y = (\sin y)^x$

(ii) $x^3 + y^3 = 3axy$

(iii) $y = (\sec x)^{\tan x}$

(b) Find the derivative of $y = e^{\sqrt{x}}$ from first principle. 3

(c) Find the slope of the curve given by $x^2 + y^2 + 2x - 4y - 20 = 0$ at (2, 6). 2

(d) Evaluate the derivative of x^7 with respect to x^4. 2

6. *(a)* Let $y = \tan^{-1} x$. Show that—

(i) $(1 + x^2)y = 1$

(ii) $(1 + x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$ 2+4=6

(b) Using L' Hospital's rule, evaluate the following (any *two*) : 3×2=6

(i) $\underset{x \to 0}{\text{Lt}} (\sin x)^{2\tan x}$

(ii) $\underset{x \to 0}{\mathrm{Lt}} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$

(iii) $\underset{x \to 0}{\mathrm{Lt}}\, x \log x$

(c) The radius of a circle is increasing at the rate of 2 cm per second. At what rate is the area increasing when the radius is 10 cm? 3

UNIT—IV

7. *(a)* Evaluate (any *one*) : 3

(i) $\int \dfrac{x^2}{(a+bx)^3} dx$

(ii) $\int \sqrt{\dfrac{x-1}{x+1}} dx$

(b) Evaluate (any *one*) : 4

(i) $\int_0^1 \dfrac{dx}{(1+x^2)^{3/2}}$

(ii) $\int_0^{\pi/2} \dfrac{\sin x dx}{\sin x + \cos x}$

(c) Using the definition of definite integral as the limit of the sum, evaluate

$$\int_0^2 (x^2+1) dx$$ 5

(d) Evaluate $\int_0^1 \dfrac{dx}{\sqrt{1-x^2}}$, if it converges. 3

8. *(a)* Show that

$$\int_0^{\pi/2} \log \sin x dx = \frac{\pi}{2} \log \frac{1}{2}$$ 4

(b) Evaluate : 5

$$\lim_{n \to \infty} \left\{ \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n} \right\}$$

(c) If

$$I_n = \int_0^{\pi/2} \sin^n x dx,$$

where n is a positive integer, $n > 1$; then prove that

$$I_n = \frac{n-1}{n} I_{n-2}$$

Hence evaluate $\int_0^{\pi/2} \sin^5 x dx$. 3+3=6

UNIT—V

9. *(a)* Show that $v = \dfrac{A}{r} + B$ satisfies the differential equation

$$\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$$ 2

(b) Solve : 5

$$\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$$

(c) Show that the equation

$$xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$$

is exact. 2

(d) Solve any *two* of the following : 3×2=6

(i) $xy^2 dy - y^3 dx + y^2 dy = dx$

(ii) $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

(iii) $\frac{dy}{dx} = \frac{6x - 2y - 7}{3x - y + 4}$

10. *(a)* Find the differential equation of the family of curves

$$y = e^x (a \cos x + b \sin x)$$

where a and b are arbitrary constants. 2

(b) Solve completely $y = px + \frac{a}{p}$ where $p = \frac{dy}{dx}$. 4

(c) Find the orthogonal trajectories of the series of hyperbolas $xy = a^2$. 3

(d) Solve any *two* of the following : 3×2=6

(i) $(D^2 - D - 2)y = e^{2x}$

(ii) $(D^2 - 8D + 15)y = 0$

(iii) $(D^3 - 1)y = 0$

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