Odd Semester, 2020

(Held in March, 2021)

STATISTICS

(Honours)

[STEH–52 (TH)]

(Linear Models, Regression and Operations Research, Design of Experiments)

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Explain the following terms : 6

(i) Contrasts

(ii) Estimation and error spaces

(iii) Best linear unbiased estimator

(b) State and prove a necessary and sufficient condition of estimability of a linear parametric function. 6

2. *(a)* Consider the Gauss-Markov linear model $y = A\beta + \varepsilon$, with all the regular assumptions. If $\lambda'\beta$ is estimable, then prove that its best estimate is $\lambda'\hat{\beta}$ where $\hat{\beta}$ is any solution of the equations $A'A\beta = A'y$. 8

(b) If the coefficient matrix A has full column rank, then obtain the variance-covariance matrix of $\hat{\beta}$. 4

UNIT—II

3. In simple linear regression, derive the maximum likelihood estimators of the parameters under the assumption of normally distributed errors. 11

4. *(a)* Describe a test procedure for testing significance of regression, in a multiple linear regression, of the response y on the regressor variables $x_1, x_2, \cdots x_k$. 7

(b) Write a note on normal probability plot of residuals. 4

UNIT—III

5. Write out the complete analysis of two-way classified data with an observation per cell under a random effect model stating clearly the assumptions used, hypothesis to be tested, test statistic to be used along with ANOVA table. 11

6. *(a)* What is confounding in factorial experiment? Distinguish between complete and partial confounding. 2+3=5

(b) Consider the setup of 2^3-factorial experiment. Suppose the block size is 2^2 and 2 replications are made such that *AB* is confounded in replicate 1 and *ABC* is confounded in replicate 2. Write down the design. 6

UNIT—IV

7. *(a)* Formulate the following linear programming problem :

A used car dealer wishes to stock up his lot to maximize his profit. He can select cars *A*, *B* and *C* which are valued at ₹ 5,000, ₹ 7,000 and ₹ 8,000 respectively. These can be sold at ₹ 6,000, ₹ 8,500 and ₹ 10,500 respectively. For each type, the probabilities of sale are :

Type of car	*Prob. of sale in 90 days*
A	0·7
B	0·8
C	0·6

For every two cars of *B*, he should buy one car of type *A* or *C*. If he has ₹ 1,00,000 to invest, what should he buy to maximize his expected gain? 8

(b) Define the following terms : 3

(i) Basic solution

(ii) Feasible solution

8. *(a)* Define slack and surplus variables in LPP with examples. 4

(b) Discuss briefly with examples the optimization problems occurring in various fields that can be formulated as LPP. 7

UNIT—V

9. *(a)* What is a transportation problem? State briefly with an example, how it can be formulated as an LPP. 5

(b) Describe a method to obtain initial basic feasible solution for a transportation problem. 6

10. *(a)* What is an assignment problem? Give its mathematical formulation. 5

(b) Write down the steps of Hungarian assignment algorithm. 6

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