

2021

(July)

COMMERCE

(Honours)

(Fundamental Mathematics)

(BC-202)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks
for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. (a) Define a matrix. Find matrices A and B when

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 2 \\ 1 & 1 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 4 & 4 \\ 4 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

2+3=5

- (b) Show that

$$\begin{vmatrix} 1 & a & 1 & 1 \\ 1 & 1 & b & 1 \\ 1 & 1 & 1 & c \end{vmatrix} = abc + 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \quad 5$$

- (c) If

$$A = \begin{pmatrix} 3 & 3 & 4 \\ 2 & 3 & 4 \\ 0 & 1 & 1 \end{pmatrix}$$

prove that $A^3 = A^{-1}$. 5

Or

- (a) Solve the following system of equations using Cramer's rule : 5

$$\begin{matrix} 2x & 3y & z & 2 \\ x & 2y & z & 1 \\ 2x & y & 6z & 4 \end{matrix}$$

- (b) Using elementary column operations, transform

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

to a diagonal matrix. 5

- (c) At a store, Tony pays ₹ 34 for 2 kgs of Apples, 1 kg of Berries and 4 kgs of Cherries. Liza pays ₹ 35 for 3 kgs of Apples, 2 kgs of Berries and 2 kgs of Cherries. Bob pays ₹ 49 for 5 kgs of Apples, 3 kgs of Berries and 2 kgs of Cherries. What is the price per kg of Apples, Berries and Cherries separately? 5

(3)

UNIT—II

2. (a) If $f(x)$ is a linear function from R to R such that $f(-1) = 5$, and $f(1) = 6$, find $f(x)$. Also find the value of $f(11)$. $3+1=4$
- (b) Find the domain and the range of the following functions : $2+4+3=9$
- (i) $\sqrt{6-x}$
- (ii) $\sqrt{8-2x-3x^2}$
- (iii) 2^x
- (c) Define a logarithmic function with an example. 2

Or

- (a) If $f(x) = \log_e(x + \sqrt{1+x^2})$ show that $f(x) + f(-x) = 0$. 4
- (b) Draw the graph of the following function :
- $$f(x) = \begin{cases} x^2 - 1, & \text{when } x < 0 \\ 1, & \text{when } x = 0 \\ x - 1, & \text{when } x > 0 \end{cases}$$
- Also find $F(-2)$ and $F(7)$. 6

(4)

- (c) The cost of hiring a catering service to serve food for a party is ₹ 150 per head for 20 persons or less, ₹ 130 per head for 21 to 50 persons and ₹ 110 per head for 51 to 100 persons. For 100 or more persons, the cost is ₹ 100 per head. If x is the number of persons, find the total cost function $C(x)$ of the catering service. If exactly 200 people attend the party, how much will the catering service earn? 5

UNIT—III

3. (a) Evaluate the following limits : $3+3=6$
- (i) $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$
- (ii) $\lim_{x \rightarrow 0} \frac{\sqrt{1-2x} - \sqrt{1-3x}}{x}$
- (b) Explain why the following limit does not exist : 3

$$\lim_{x \rightarrow 0} \frac{x}{|x|}$$

(5)

(c) A function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} \frac{1}{2}x, & \text{when } 0 \leq x \leq \frac{1}{2} \\ \frac{1}{2}, & \text{when } x = \frac{1}{2} \\ \frac{3}{2}x, & \text{when } \frac{1}{2} < x \leq 1 \end{cases}$$

Show that $f(x)$ is discontinuous at $x = \frac{1}{2}$.

Is $f(x)$ continuous at $x = \frac{4}{7}$? Justify your

answer.

6

Or

(a) Find the first-order derivative of the following functions : 3+3+3=9

(i) $y = (x^2 - 2)^5(3x^4 - 5)^4$

(ii) $y = \frac{e^x \log_e x}{x^2}$

(iii) $y = x^{\log_e x}$

(b) Using the first principle, find the first-order derivative of the following function :

3

$$y = \frac{1}{\sqrt{x^2 - 2^2}}$$

(c) If the rate of change of y with respect to x is 5 and x is changing at 3 units per second, how fast is y changing?

3

(6)

UNIT—IV

4. (a) A company charges ₹550 for a transistor set on orders of 50 or less sets. The charge is reduced by ₹5 per set for each set ordered in excess of 50. Find the largest size order that the company should allow in order to receive a maximum revenue. Also find the maximum revenue.

6

(b) The manufacturing cost of an item consists of ₹1,200 as overhead cost, material cost of ₹4 per item and the labour cost of ₹ $3x^2/256$ for x items produced. How many items should be produced in order to have minimum average cost?

5

(c) Find the maximum and the minimum values of $f(x) = 2x^2 - x^3$.

4

Or

(a) The side of an equilateral triangle is 5 cm and is increasing at the rate of $\sqrt{3}$ cm/s. How fast is its area increasing?

4

(b) If the demand function is $p = 4 - 5x^2$, for what value of x , the elasticity of demand will be unitary? Here, p is price per unit of output and x is the output.

3

(7)

- (c) Find the elasticities of demand and supply at equilibrium price for demand function $p = \sqrt{100 - x^2}$ and supply function $x = 2p - 10$. 8

UNIT—V

5. (a) Mr. X plans to invest ₹5,000 for 3 years. He may choose to invest the amount at 9% per annum compound interest accruing at the end of each quarter of the year or he may choose to invest it at 9.5% per annum compound interest accruing at the end of each year. Which investment will he choose for better returns? 5
- (b) A bond is available for ₹1,500 it offers, including one immediate payment and 10 annual payments of ₹200. Find the rate of return on the bond. 5
- (c) What annual instalment should be paid to clear the debt of ₹645 in 4 years at the rate of 5% per annum simple interest? 5

(8)

Or

- (a) By investing ₹8,000 in the shares of a company, Peter gets an income of ₹200 when the dividend is 10%. If the initial value of each share is ₹80, find the market value of each share. 5
- (b) A machine costs ₹5,00,000 with a working life of 5 years and a scrap value of ₹1,00,000 at the end. Calculate its rate of depreciation. 5
- (c) A bond presently sells at ₹112 which carries a coupon rate of 8% per annum. If your expected rate of return is 10%, would you like to purchase the bond? 5

★ ★ ★