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(July)

COMMERCE

(Honours)

(Fundamental Mathematics)

(BC–202)

(Under Revised Syllabus)

Marks : 75

Time : 3 hours

The Figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Using elementary row operations method, find out A^{-1} when

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 5 & 2 & 7 \\ 4 & 3 & 1 \end{bmatrix}$$

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(b) Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

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2. *(a)* S Ltd. produces two products X and Y. Each product is first processed on machine M_1 and then on M_2. Each unit of X requires 20 minutes time on M_1 and 10 minutes time on M_2 while the corresponding times for Y are 10 minutes on M_1 and 20 minutes on M_2. The total available time for each machine is 600 minutes. Calculate the number of units of X and Y by constructing a matrix equation of the form $AX = B$ and then solving by matrix inversion method. 5

(b) State any five properties of determinants. 5

(c) Solve

$x + 3z + 1 = 0; 2x + y + 4z + 2 = 0$ and

$y + 2z + 4 = 0$ using Cramer's rule. 5

(3)

UNIT—II

3. *(a)* If α, β be the roots of the equation

$$ax^2 + bx + c = 0$$

then find the values of—

 (i) $(\alpha + \beta)^2$

 (ii) $\alpha^2 + \beta^2$ 5

(b) The total profit y of P Ltd. from manufacturing and sale of x units of toys is given by

$$y = \frac{x^2}{400} + 2x + 80$$

Find—

  (i) how many toys the P Ltd. must sell per period to earn maximum profit;

  (ii) the profit per toy when the maximum is achieved.   5

(c) A firm produces an output of x tons of a certain product at a total cost given by $C = x^3 + 4x^2 + 7x$. Find at what level of output, average cost is the minimum and what level will it be.   5

4. *(a)* Solve :   5

$$(x+1)(x+2)(x+3)(x+4) = 120$$

(b) The pricing policy of H Ltd. follows the demand function $p = D(x)$, where $D(x)$ is the price per unit when x units are demanded. After studying the market trends the H Ltd. determines the price function that is given by

$$D(x) = 2000 - 4x$$

If the product is to be marketed, H Ltd. will incur a fixed cost of ₹ 60,000 and will have to pay ₹ 600 for each unit that is produced and placed in the store. At what sales level, H Ltd. is expected to recover its costs?   5

(c) The total cost of producing 10 units is ₹ 120 and that of 30 units is ₹ 160. Assuming the cost function is linear, find—

  (i) the marginal cost;

  (ii) the cost function;

  (iii) the fixed cost;

  (iv) the total cost of producing 50 units.   5

UNIT—III

5. *(a)* State the laws of limits. 9

(b) Evaluate :

$$\lim_{n} \ \frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \cdots + \frac{n}{n^2}$$ 2

(c) Given $f(x) = ax^2 + bx + c$, show that

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 2ax + b$$ 4

6. Differentiate the following w.r.t. x : 4+4+3+4=15

(a) $\sqrt{\dfrac{1+x}{1+x}}$

(b) $2x^2 + 3xy + 5y^2 = 0$

(c) $y = 5 e^{x^2+2}$

(d) $y = x^{x^2}$

UNIT—IV

7. *(a)* Find the maximum and minimum values of

$$\frac{x^2 + 3x + 4}{x + 8}$$ 7

(b) The rate of change of total cost y of a commodity per unit change of output is called the marginal cost of it. If there exists a relationship between y and x in the form

$$y = \frac{3x(x+7)}{x+5} + 5$$

prove that marginal cost falls continuously as the output increases. 4

(c) If q be the number of workers employed, the average cost of production is given by

$$C = \frac{3}{2(q+4)} + 24q$$

Show that $q = 4\cdot 25$ will make the expression minimum. In the interest of the management, will you advise to employ 4 or 5 workers? 4

8. *(a)* Find the maximum and minimum values of the function

$$f(x) = x^3 + 2x^2 + x + 9$$ 7

(b) The total cost of daily output of q tons of coal is

$$₹ \; \frac{1}{10}q^3 \; 3q^2 \; 50q$$

What is the value of q when average cost is the minimum? Also verify at this level that average cost equals marginal cost. 4

(c) The material demanded is 10000 units per year, the material cost is ₹ 1 per unit and ₹ 25 as cost to make the factory ready per production run regardless of the number of output produced in units, and the storing cost is 12·50% p.a. of value of average inventory $\frac{x}{2}$. Find—

  (i) total cost function;

  (ii) economic order quantity (EOQ) and total cost corresponding to EOQ. 4

UNIT—V

9. *(a)* A truck purchased at a cost of ₹ 60,000 depreciates at a rate of 10% p.a. and its maintenance cost for the first year is ₹ 2,000 which increases at 2% every year. If the scrap value realised when

sold is ₹ 35,429·40, find the minimum average annual return from the truck, the owner must earn so as not to sustain any loss. 5

(b) ₹ 2,00,000 is invested at an annual rate of interest at 10%. What is the amount after 2 years, if the compounding is done—

  (i) annually;

  (ii) semi-annually;

  (iii) monthly;

  (iv) daily? 5

(c) An investor intends to purchase a 3-year ₹ 1,000 bond with nominal interest rate at 10%. At what price the bond may be purchased now, if it matures at per and he requires a rate of return at 14%? 5

10. *(a)* A man took a loan of ₹ 12,000 from a bank. He paid 5% interest on a part of the loan and 4% on the remainder. After 3 years, he has paid ₹ 1,650 as total interest. Find the principal at 4% interest. 5

(b) A sum of money doubles itself in 50 years at a certain rate percent of simple interest. How long will it take to double itself at the same rate of interest compounded annually? 5

(**9**)

(c) A machine depreciates in value each year at 10% of its previous value and at the end of the fourth year its value is ₹ 1,31,220. Find the original value of the machine. 5

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