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(July)

MATHEMATICS

(Elective/Honours)

(Algebra—II and Dynamics)

[GHS–41]

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

SECTION—A

(Algebra—II)

UNIT—I

1. *(a)* Prove that the set $\{1, \quad 1, i, \quad i\}$ is a finite Abelian group of order 4 with respect to multiplication $[i^2 \quad\; 1]$. 3

(b) State and prove Lagrange's theorem. 1+4=5

(c) Show that every subgroup of a cyclic group is cyclic. 4

(d) Show that two right cosets Ha, Hb are distinct iff two left cosets $a \; ^1H$, $b \; ^1H$ are distinct. 3

2. *(a)* Verify whether the binary operation ‘ ’ defined on $\mathbb{Q}$ by $a \quad b \quad ab/2$ is *(i)* commutative and *(ii)* associative. 1+2=3

(b) Show that the equations $ax \quad b$ and $ya \quad b$ have unique solutions in a group G, where $a, b \quad G$. 3

(c) Show that a group of order less than 6 is always Abelian. 5

(d) Show that a group G is Abelian if and only if $(ab)^2 \quad a^2b^2$ for all $a, b \quad G$. 4

UNIT—II

3. *(a)* Find the range of values of k for which the roots of the equation $x^4 \quad 4x^3 \quad 8x^2 \quad k \quad 0$ are all real. 5

(b) Solve $x^4 \quad x^3 \quad 3x^2 \quad 31x \quad 26 \quad 0$, if one of the roots of the given equation is $2 \quad 3i$. 5

(c) Remove the second-term of the equation $x^3 \quad 6x^2 \quad 12x \quad 19 \quad 0$ and then solve the resulting equation. 5

4. *(a)* Express $2x^4 \quad 4x^3 \quad 5x^2 \quad x \quad 7$ as a function of $(x \quad 2)$. 4

(b) Solve the equation $x^3 \quad 18x \quad 35 \quad 0$ by Cardan's method. 5

(c) If $\ ,\ ,\ $ are the roots of the equation $x^3 \quad px^2 \quad qx \quad r \quad 0$ form the equation whose roots are $\frac{1}{\ }, \quad \frac{1}{\ }, \quad \frac{1}{\ }.$ 4

(d) Find the three values of $(1 \quad i)^{1/3}$. 2

SECTION—B

(Dynamics)

UNIT—III

5. *(a)* A particle moves with SHM in a straight line. In the first second after starting from rest it travels a distance a, in the next second it travels a distance b in the same direction. Prove that the amplitude of motion is

$$\frac{2a^2}{3a \quad b}$$

and its period is

$$\frac{2}{\cos^{\ 1} \frac{b \quad a}{2a}}$$

2½+2½=5

(b) A sphere impinges obliquely on another sphere at rest. If the two spheres are smooth, perfectly elastic and equal in mass, prove that they move at right angles to each other after impact. 6

(c) A particle is projected upwards from the earth's surface with a velocity 1 km/sec. Considering variations in gravity, find roughly the greatest height attained. 4

6. *(a)* A particle is moving in an SHM of amplitude a. Find the new amplitude if the velocity were doubled when the particle is at a distance $a/2$ from the centre, the period remaining unaltered. 4

(b) A particle is moving under the acceleration $\ /x^2$ towards the origin, where x is its distance from the attracting centre. If it starts from rest at a distance a, show that the time occupied from $x \quad \frac{3a}{4}$ to $x \quad \frac{a}{4}$ is one-third of the time taken from $x \quad a$ to $x \quad 0$. 5

(c) Two spheres of masses m, M impinge directly when moving in opposite directions with velocities u, v respectively. If the sphere of mass m is brought to rest by the collision, show that $v(m \quad eM) \quad M(1 \quad e)u$. After the collision the sphere of mass M is

acted on by a constant retarding force which brings it to rest after travelling a distance a. Prove that the magnitude of this force is $\frac{Me^2(u \quad v)^2}{2a}$. 3+3=6

UNIT—IV

7. *(a)* A projectile aimed at a mark, which is in the horizontal plane through the point of projection, falls a metre short of it when the elevation is and goes b metre too far when the elevation is . Show that, if the velocity of projection be the same in all the cases, the proper elevation is

$$\frac{1}{2}\sin^{\quad 1}\frac{a\sin 2 \quad b\sin 2}{a \quad b}$$ 5

(b) A particle, of mass m, is projected vertically under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is

$$\frac{V^2}{g}[\quad \log(1 \quad)]$$

where V is the terminal velocity of the particle and V is its initial vertical velocity. Show that the corresponding time is $\frac{V}{g}\log(1 \quad)$. 3+3=6

(c) A particle moves towards a centre of attraction starting from rest at a distance a from the centre. If its velocity, when at any distance x from the centre, varies as $\sqrt{\frac{a^2 \quad x^2}{x^2}}$, find the law of force. 4

8. *(a)* The direction of motion of a projectile at a certain instant is inclined at an angle to the horizontal; after t seconds it is inclined at an angle . Prove that the horizontal component of the velocity is $\frac{gt}{\tan \quad \tan}$. 5

(b) If the resistance per unit mass is kv^2 and the particle slides on a smooth straight wire inclined at an angle to the vertical, prove that the space s described in time t from rest is given by $e^{ks} \quad \frac{1}{2}(e^{bt} \quad e^{\quad bt})$, where $b^2 \quad kg\cos$. 5

(c) A particle moves in a straight line under a force to a point in it varying as (distance) $^{4/3}$. Show that the velocity in falling from rest at infinity to a distance a is equal to that acquired in falling from rest at distance a to a distance $\frac{a}{8}$. 5

UNIT—V

9. *(a)* If the tangential and normal acceleration of a particle describing a plane curve be constant throughout, prove that—

(i) the radius of curvature at any time t is given by $(at \quad b)^2$;

(ii) the angle through which the direction of motion turns in time t is given by $A \log (1 \quad Bt)$. 2+3=5

(b) A particle is fastened to a point by an inelastic string of length a and hangs vertically. It is projected horizontally with a velocity $\sqrt{6ag}$ and describes a vertical circle. Show that the tension in the string when it is in a horizontal position is four times the tension when the particle is at the highest point. 5

(c) A shot of mass m is fired from a gun of mass M with a velocity u relative to the gun. Show that the actual velocities of the shot and the gun are $\frac{Mu}{M \quad m}$ and $\frac{mu}{M \quad m}$ respectively and that their kinetic energies are inversely proportional to their masses. 5

10. *(a)* A mass m after falling through a height h begins to raise a mass M greater than itself and connected with it by means of an inextensible string over a fixed smooth pulley. Show that M will have returned to its original position at the end of time $\frac{2m}{M \quad m}\sqrt{\frac{2h}{g}}$ after rising through a height $\frac{m^2 h}{M^2 \quad m^2}$. 6

(b) Show that for a particle, sliding down the arc and starting from a cusp of a smooth cycloid whose axis is vertical and vertex lowest, the vertical velocity is maximum when it has described half the vertical height. 4

(c) A particle is projected along the inside of a smooth vertical circle of radius a from the lowest point. Show that the velocity of projection required in order that after leaving the circle the particle may pass through the centre is $\sqrt{\frac{1}{2}ag}\,(\sqrt{3} \quad 1)$. 5

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