2/EH–29 (ii) (Syllabus–2015)

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(July)

MATHEMATICS

(Elective/Honours)

(Geometry and Vector Calculus)

(GHS–21)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. *(a)* Transform the equation

$$3x^2 \quad 8xy \quad 3y^2 \quad 2x \quad 2y \quad 2 \quad a^2$$

referred to new axes through (1, 1) rotated through an angle $\frac{}{4}$. 5

(b) Prove that the straight lines represented by the equation

$$ax^2 \quad 2hxy \quad by^2 \quad 2gx \quad 2fy \quad c \quad 0$$

will be equidistant from the origin if $f^4 \quad g^4 \quad c(bf^2 \quad ag^2)$. 6

(c) Find the equation of the polar of the origin with respect to the conic

$$ax^2 \quad 2hxy \quad by^2 \quad 2gx \quad 2fy \quad c \quad 0$$ 4

2. *(a)* Prove that the pair of lines joining the origin to the points of intersection of the curve $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ by the line $lx \quad my \quad n \quad 0$ are coincident if $a^2l^2 \quad b^2m^2 \quad n^2$. 5

(b) Reduce the equation

$$4x^2 \quad 4xy \quad y^2 \quad 2x \quad 26y \quad 9 \quad 0$$

to the standard form. 6

(c) Find the equation of the diameter of the conic $4x^2 \quad 6xy \quad 5y^2 \quad 1$, conjugate to the diameter $y \quad 2x \quad 0$. 4

UNIT—II

3. *(a)* Prove that the locus of the foot of the perpendicular from either focus upon any tangent to the ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ is a circle. 6

(b) Obtain the equation of the asymptotes of the hyperbola $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$. Hence show that $a \quad b$ if the hyperbola $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ is rectangular. What is the value of the eccentricity of the rectangular hyperbola? 2+2+1=5

(c) The normal at the point $(at_1^2, 2at_1)$ meets the parabola again at the point $(at_2^2, 2at_2)$. Prove that $t_2 \quad t_1 \quad 2/t_1$. 4

4. (a) Prove that the semi-latus rectum of any conic is the harmonic mean between the segments of any focal chord. 5

(b) Show that the normal to the rectangular hyperbola $xy \quad c^2$ at the point t meets the curve again at the point t such that $t^3 t \quad 1$. 5

(c) Show that the locus of the poles of normal chords of the parabola $y^2 \quad 4ax$ is $(x \quad 2a)y^2 \quad 4a^3 \quad 0$. 5

UNIT—III

5. (a) Find the coordinates of the point where the join of (2, 3, 1) and (1, 2, 4) cuts the plane $2x \quad 3y \quad 5z \quad 3 \quad 0$. 5

(b) Find the equation of the plane which contains the line of intersection of the plane $x \quad 2y \quad 3z \quad 4 \quad 0$ and $2x \quad y \quad z \quad 5 \quad 0$ and which is perpendicular to the plane $5x \quad 3y \quad 6z \quad 8 \quad 0$. 5

(c) Find the equation of the cone, whose vertex is (, ,) and the base is the parabola $z \quad 0, y^2 \quad 4ax$. 5

6. (a) Find the equation of the cylinder generated by the lines parallel to the line $\frac{x}{1} \quad \frac{y}{2} \quad \frac{z}{1}$ and intersecting the guiding curve $z \quad 3, x^2 \quad y^2 \quad 4$. 5

(b) Find the equation of the sphere which passes through the points (3, 0, 0), (0, 1, 0), (0, 0, 2) and whose centre lies on the plane $3x \quad 2y \quad 4z \quad 1 \quad 0$. 5

(c) Prove that the two lines

$$\frac{x \quad 1}{1} \quad \frac{y \quad 1}{2} \quad \frac{z \quad 1}{3}$$

and $\frac{x \quad 4}{2} \quad \frac{y \quad 6}{3} \quad \frac{z \quad 8}{4}$ are coplanar and find their point of intersection. 5

UNIT—IV

7. *(a)* Reduce the expression

$$(\vec{b} \quad \vec{c}) \; \{(\vec{c} \quad \vec{a}) \quad (\vec{a} \quad \vec{b})\}$$

in its simplest form and prove that it vanishes when $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar. 5

(b) Show that

$$\hat{i} \quad (\vec{a} \quad \hat{i}) \quad \hat{j} \quad (\vec{a} \quad \hat{j}) \quad \hat{k} \quad (\vec{a} \quad \hat{k}) \quad 2\vec{a}$$ 4

(c) If $\vec{r} \quad a\cos t\hat{i} \quad a\sin t\hat{j} \quad at\tan \quad \hat{k}$, find

(i) $\left|\dfrac{d\vec{r}}{dt} \quad \dfrac{d^2\vec{r}}{dt^2}\right|$

(ii) $\dfrac{d\vec{r}}{dt}, \dfrac{d^2\vec{r}}{dt^2}, \dfrac{d^3\vec{r}}{dt^3}$ 3+3=6

8. *(a)* Show that the necessary and sufficient condition for the vector $\vec{v}(t)$ to have a constant direction is

$$\vec{v} \quad \frac{d\vec{v}}{dt} \quad \vec{0}$$ 5

(b) Show that the four points $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are coplanar if and only if

$$[\vec{b}, \vec{c}, \vec{d}] \quad [\vec{c}, \vec{a}, \vec{d}] \quad [\vec{a}, \vec{b}, \vec{d}] \quad [\vec{a}, \vec{b}, \vec{c}]$$ 5

(c) Find the area of the triangle *OAB* formed by the two points *A* and *B* whose position vectors are $\hat{i} \quad 2\hat{j} \quad 3\hat{k}$ and $3\hat{i} \quad 2\hat{j} \quad \hat{k}$ respectively; *O* being the origin. 5

UNIT—V

9. *(a)* A particle moves along the curve $x \quad e^{\;t}$, $y \quad 2\cos 3t$ and $z \quad 2\sin 3t$. Determine the velocity and acceleration at any time t and their magnitudes at $t \quad 0$. 2+1+2=5

(b) Find the directional derivative of the function $xy^2 \quad yz^2 \quad zx^2$ along the tangent to the curve $x \quad t$, $y \quad t^2$ and $z \quad t^3$ at the point (1, 1, 1). 5

(c) Show that

(i) grad $(\vec{r} \quad \vec{a}) \quad \vec{a}$

(ii) grad $[\vec{r}, \vec{a}, \vec{b}] \quad \vec{a} \quad \vec{b}$

where $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ and $\vec{a}$, $\vec{b}$ are constant vectors. 3+2=5

10. *(a)* Find the equation of the tangent plane to the surface $z = x^2 + y^2$ at the point $(1, -1, 2)$. 4

(b) Define curl and divergence of a vector valued function $\vec{F}$. Show that $\operatorname{div}(\operatorname{curl}\vec{F}) = 0$. 1+1+3=5

(c) Let $\vec{A}$ and $\vec{B}$ be vector point functions. Then prove that

$$\operatorname{div}(\vec{A} \times \vec{B}) = \vec{B} \cdot \operatorname{curl}\vec{A} - \vec{A} \cdot \operatorname{curl}\vec{B}$$

Also, if $\vec{A}$ and $\vec{B}$ are irrotational, then prove that $\vec{A} \times \vec{B}$ is solenoidal. 4+2=6

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