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(July)

MATHEMATICS

(Elective/Honours)

(Geometry and Vector Calculus)

(GHS–21)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. *(a)* If by rotation of the rectangular axes about the origin, the expression $ax^2 \quad 2hxy \quad by^2$ changes to $a x^{\ 2} \quad b y^{\ 2} \quad 2h x y$, then prove that $a \quad b \quad a \quad b$ and $ab \quad h^2 \quad a b \quad h^{\ 2}$. 5

(b) Reduce the equation

$$9x^2 \quad 24xy \quad 16y^2 \quad 18x \quad 101y \quad 19 \quad 0$$

to standard form and find the respective conic. 5

(c) Show that the equation

$$6x^2 \quad 11xy \quad 10y^2 \quad 35y \quad 14x \quad 0$$

represents a pair of straight lines and find the distance between them. 5

2. *(a)* Find the equations of the asymptotes to the conic $2xy \quad 5x \quad 2y \quad 9 \quad 0$. 4

(b) Find the diameter of the conic

$$15x^2 \quad 20xy \quad 16y^2 \quad 1$$

conjugate to the diameter $y \quad 2x \quad 0$. 4

(c) Find the equation of the tangent to the conic $4x^2 \quad 3xy \quad 2y^2 \quad 3x \quad 5y \quad 7 \quad 0$ at the point $(1, \quad 2)$. 4

(d) Prove that the points $(1, \ 2)$ and $(\ 2, \ 3)$ are conjugates with respect to the conic

$$2x^2 \quad 6xy \quad y^2 \quad 4x \quad 2y \quad 8 \quad 0$$ 3

UNIT—II

3. *(a)* Find the equation of the parabola whose focus is the point $(\ 1, 1)$ and whose directrix is the straight line $x \quad y \quad 1 \quad 0$. Find also the length of the latus rectum. 5

(b) If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ be the extremities of a focal chord of a parabola $y^2 \quad 4ax$, then prove that $t_1 t_2 \quad 1$. 5

(c) Show that the condition that the line $y \quad mx \quad c$ is a tangent to the ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$ is $c^2 \quad a^2m^2 \quad b^2$. 5

4. *(a)* Prove that the chords of a hyperbola which touch the conjugate hyperbola are bisected at the point of contact. 5

(b) Show that the locus of the point of intersection of tangents at two points on the ellipse $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$, where the difference of their eccentric angle is 2 , is $\frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad \sec^2$. 5

(c) Show that the locus of the poles of normal chords of the parabola $y^2 \quad 4ax$ is $(x \quad 2a)y^2 \quad 4a^3 \quad 0$. 5

UNIT—III

5. *(a)* Find the length and equations of the line of shortest distance between the lines

$$\frac{x \quad 3}{4} \quad \frac{y \quad 6}{3} \quad \frac{z}{2} \text{ and } \frac{x \quad 2}{4} \quad \frac{y}{1} \quad \frac{z \quad 7}{1}$$ 5

(b) Prove that the lines

$$\frac{x \quad 1}{1} \quad \frac{y \quad 1}{2} \quad \frac{z \quad 1}{3} \text{ and } \frac{x \quad 4}{2} \quad \frac{y \quad 4}{3} \quad \frac{z \quad 8}{4}$$

are coplanar and find their point of intersection. 5

(c) Find the equation of the cone whose vertex is the origin and base the circle $x \quad a,\ y^2 \quad z^2 \quad b^2$. 5

6. *(a)* Find the equation of the right circular cylinder whose guiding curve is $x^2 \quad y^2 \quad z^2 \quad 9,\ x \quad y \quad z \quad 3$ and z-axis as its generator. 5

(b) Find the equation of the sphere which touches the sphere $x^2 \quad y^2 \quad z^2 \quad 21$ at $(1, \quad 2, 4)$ and passes through the point $(3, 4, 0)$. 5

(c) Find the radius of the circle where the plane $x \quad 2y \quad 2z \quad 3$ intersects the sphere

$$x^2 \quad y^2 \quad z^2 \quad 8x \quad 4y \quad 8z \quad 45$$ 5

Unit—IV

7. *(a)* For any three non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ show that

$$[\vec{a} \quad \vec{b}, \vec{b} \quad \vec{c}, \vec{c} \quad \vec{a}] \quad 2[\vec{a}, \vec{b}, \vec{c}]$$ 5

(b) If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the points A, B, C, then show that the vector $\vec{a} \quad \vec{b} \quad \vec{b} \quad \vec{c} \quad \vec{c} \quad \vec{a}$ is perpendicular to the plane ABC. 5

(c) Prove that the necessary and sufficient condition that a proper vector $\vec{u}(t)$ has constant magnitude is that $\vec{u} \quad \frac{d\vec{u}}{dt} \quad 0.$ 5

8. *(a)* If $\vec{r} \quad 5t^2\hat{i} \quad t\hat{j} \quad t^3\hat{k}$ and $\vec{s} \quad \sin t\,\hat{i} \quad \cos t\,\hat{j}$, then find the values of $\frac{d}{dt}(\vec{r} \quad \vec{s})$ and $\frac{d}{dt}(\vec{r} \quad \vec{s})$. 5

(b) If $\vec{r} \quad \cos \quad t\hat{i} \quad \sin \quad t\hat{j}$, where is a constant, then show that $\vec{r} \quad \frac{d\vec{r}}{dt}$ is a constant vector. 5

(c) Show that

$$(\vec{a} \quad \vec{b}) \quad \vec{c} \quad \vec{a} \quad (\vec{b} \quad \vec{c})$$

when and only when $(\vec{c} \quad \vec{a}) \quad \vec{b} \quad 0$. 5

Unit—V

9. *(a)* Find the directional derivative of the function $f(x, y, z) \quad x^2yz \quad 4xz^2$ at the point (1, 2, 1) in the direction of the vector $2i \quad j \quad 2k$. 5

(b) Prove that grad $r^m \quad mr^{m \quad 2}\vec{r}$ where $r \quad |\vec{r}|$. 5

(c) If $\vec{F} \quad (x \quad y \quad 1)\hat{i} \quad \hat{j} \quad (x \quad y)\hat{k}$, then prove that $\vec{F} \quad \text{curl}\,\vec{F} \quad 0$. 5

10. *(a)* Prove that div $\frac{\vec{r}}{r^3} \quad 0$, where $r \quad |\vec{r}|$. 5

(b) If $\vec{f} \quad x^2y\hat{i} \quad 2xz\hat{j} \quad 2yz\hat{k}$, then find div $\vec{f}$ and curl $\vec{f}$. 2+3=5

(c) *(i)* Determine a so that the vector

$$\vec{v} \quad (2x \quad 3y)\hat{i} \quad (3ay \quad z)\hat{j} \quad (x \quad 4z)\hat{k}$$

becomes solenoidal. 2

(ii) If $u \quad x^2 \quad y^2 \quad 4z$, then show that $^2u \quad 0$. 3

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