6/H–28 (vii) (Syllabus–2015)

2 0 2 1

(July)

STATISTICS

(Honours)

(Statistical Inference)

[STH–61(TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Define MVUE. If T_1 and T_2 are minimum variance unbiased estimators of ( ), then show that $T_1 \quad T_2$ almost surely. 2+4=6

(b) State and prove the Cramer-Rao inequality. 6

2. *(a)* Define consistent estimator. Show that the sample mean $\bar{x}$ based on a random sample of size n from $N(\ , \ ^2)$ is a consistent estimator of the population mean . 2+4=6

(b) Define sufficient statistic. Show that if ^{2} is known in a random sample from a normal population, the sample mean $\bar{X}$ is a sufficient statistic for mean . 2+4=6

UNIT—II

3. *(a)* Define likelihood function and state the regularity conditions for maximum likelihood estimators to be consistent and asymptotically normal. 1+2=3

(b) Find the maximum likelihood estimate for the parameter of a Poisson distribution on the basis of a random sample of size n. Also find its variance. 3

(c) Explain the method of moments for estimating parameters. What are the properties of the estimates obtained by this method? 2+3=5

4. *(a)* Discuss the concept of interval estimation and provide suitable illustration. 5

(b) Obtain the 100(1)% confidence intervals for the parameter of the normal distribution

$$f(x, \; , \;) \; \frac{1}{\sqrt{2}} e^{\frac{1}{2}\left(\frac{x}{\;}\right)^2}, \qquad x$$

6

UNIT—III

5. *(a)* Define the following : 1×6=6

(i) Statistical hypothesis

(ii) Test of significance

(iii) Null hypothesis

(iv) Critical region

(v) Size of the test

(vi) Power of the test

(b) If x 1 is the critical region for testing H_0: 2 against the alternative H_1: 1, on the basis of the single-observation from the population

$$f(x, \;) \quad e^{(\; x)}, \quad 0 \quad x$$

compute the size of the test and the power of the test. 5

6. *(a)* Explain the following terms : 2+2+2=6

(i) Most powerful test

(ii) Uniformly most powerful test

(iii) Unbiased test

(b) Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\; , \;)$ where is known. Obtain the MP test for testing H_0: $_0$ against H_1 : $_1$($_1$ $_0$). 5

UNIT—IV

7. *(a)* State the Neyman-Pearson lemma. What are its differences from the likelihood ratio test? 2+3=5

(b) Construct the likelihood ratio test for testing H_0 : $_0$ versus H_1: based on a sample of size n from $N(\; , \; ^2)$, ^{2} 0. 6

8. *(a)* Define OC function and ASN function of SPRT. 2+2=4

(b) Let X have the distribution

$$f(x, \quad) \qquad {}^{x}(1 \qquad)^{1 \quad x}, \; x \quad 0, \; 1$$

$$0 \qquad 1$$

For testing $H_0 :$ $\quad_0$ against $H_1 :$ $\quad_1$ construct the SPRT and obtain its OC function. 3+4=7

UNIT—V

9. *(a)* Differentiate between large sample and small sample tests and discuss their consequences in testing of hypothesis problems. How does the central limit theorem help in deriving large sample tests? 2+2+2=6

(b) Describe the large sample test of significance for single-binomial proportion. Also write down the confidence interval for the proportion. 4+1=5

10. *(a)* Obtain the test of significance for single mean from normal population with mean $\quad$ and variance $\quad^2$. Hence write down the related confidence interval. 3+2=5

(b) Derive the test statistic (for large samples) for the test of significance for difference of means. Also write down the related confidence interval. 4+2=6

★ ★ ★