6/H–29 (viii) (c) (Syllabus–2015)

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(July)

MATHEMATICS

(Honours)

(Hydromechanics)

(HOPT–62 : OP3)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. *(a)* Explain what you mean by equation of continuity. Derive the equation of continuity by Euler's method. 2+6=8

(b) Prove that liquid motion is possible when velocity at a point (x, y, z) is given by

$$u \quad \frac{3x^2 \quad r^2}{r^5}, v \quad \frac{xy}{r^5}, w \quad \frac{3xz}{r^5}$$

where $r^2 \quad x^2 \quad y^2 \quad z^2$, and the streamlines are the intersection of the surfaces $(x^2 \quad y^2 \quad z^2)^3 \quad c(y^2 \quad z^2)^2$, by the planes passing through x-axis. 7

2. *(a)* Show that the equation of continuity of every particle moving on the surface of a sphere is

$$\frac{\quad}{t}\cos \quad \frac{\quad}{\quad}(\quad \cos\ ) \quad \frac{\quad}{\quad}(\quad \cos\ ) \quad 0$$

being the density, , the latitude and longitude of any element and , the angular velocities of the element in latitude and longitude respectively. 7

(b) Define streamline motion and turbulent motion of a fluid. Are streamlines and path lines of particle of a fluid always the same? Justify your answer. 4

(c) Show that the ellipsoid

$$\frac{x^2}{a^2k^2t^{2n}} \quad kt^n \quad \frac{y^2}{b^2} \quad \frac{z^2}{c^2} \quad 1$$

is a possible form of the boundary surface of a liquid. 4

UNIT—II

3. *(a)* A sphere of radius a is surrounded by infinite liquid of density , the pressure at infinity being $\overline{w}$ the sphere is suddenly annihilated. Show that the pressure at a distance r from the centre immediately falls to $\overline{w}\,(1 \quad \frac{a}{r})$. 8

(b) Show that the impulsive pressure of a fluid satisfies Laplace's equation. 7

4. *(a)* Show that a steady motion of a fluid of density and velocity q under an external force derivable from velocity potential satisfies the equation

$$\frac{dp}{\quad} \quad \frac{1}{2}q^2 \qquad C$$

where C is a constant. 7

(b) An incompressible fluid is contained within the region bounded by two concentric rigid spherical surfaces of radii $R_1, R_2, (R_2 \quad R_1)$. The fluid is initially at rest. If the inner surface is now given a sudden velocity $u\vec{i}$, where $\vec{i}$ is a constant unit vector, show that the impulsive thrust on the outer surface is

$$2 \quad R_1^3 R_2^3 (R_2^3 \quad R_1^3)^{\ 1} u\vec{i}$$

where is the fluid density. 8

UNIT—III

5. *(a)* Show that in the motion of a fluid in two dimensions, if the coordinates (x, y) of an element at any time be expressed in terms of the initial co-ordinates (,) and the time, the motion is irrotational if

$$\frac{(u, x)}{(\ , \ )} \quad \frac{(v, y)}{(\ , \ )} \quad 0$$

where u and v are velocity components. 7

(b) In a fluid motion, a doublet of strength makes an angle with the x-axis. Show that the complex potential is given by $\frac{e^{i}}{z}$. 6

(c) Define the terms 'source' and 'sink'. 2

6. *(a)* Find Stokes' stream function for a fluid in motion for a uniform line source of strength m. 7

(b) Let the coordinates of two doublets of strength m_1 and m_2 situated at A and B be $(0, 0, c_1)$ and $(0, 0, c_2)$ respectively and their axes being directed towards and away from the origin respectively. Show that

$$\frac{m_2}{m_1} \quad \frac{c_2}{c_1}^{3/2}$$

if there is no transport of fluid over the surface of the sphere $x^2 \quad y^2 \quad z^2 \quad c_1 c_2$. 8

UNIT—IV

7. *(a)* Discuss the equilibrium of a body floating in more than one liquid. 4

(b) Let the forces per unit mass acting on an element of fluid at the point (x, y, z) parallel to axes be proportional to

$$y^2 \quad 2\ yz \quad z^2, z^2 \quad 2\ zx \quad x^2,$$
$$x^2 \quad 2\ xy \quad y^2.$$

Show that the equilibrium is possible if
$2 \quad 2 \quad 2 \quad 1.$ 5

(c) A small uniform circular tube whose plane is vertical contains equal quantities of fluids of densities and , (, which do not mix. If they together fill half the tube, show that the radius passing through the common surface makes with the vertical an angle $\tan^{1}$ ——. 6

8. *(a)* Find the condition for equilibrium of a fluid under pressure. 5

(b) A thin uniform rod of weight W has a particle of weight w attached to one end. It is floating in an inclined position in water with this end immersed, Show that the length of the rod above water is $\frac{w}{w \quad W}$ times the length of the rod. 5

(c) A homogeneous circular cylinder of length h, radius a and specific gravity s, floats in water. Show that the position with the axis vertical is stable if $\frac{a^2}{h^2} \quad 2s(1 \quad s)$. 5

UNIT—V

9. *(a)* A cone full of water is placed on a horizontal table. Find the thrust on its base. 4

(b) A hemispherical bowl is filled with water. Show that the horizontal fluid thrust on one half of the surface divided by a vertical diametrical plane is — times the magnitude of the resultant fluid thrust on the whole surface. 6

(c) Find the centre of pressure of a circular area of radius a immersed in a fluid depth of whose centre is h. 5

10. *(a)* Find the condition for equilibrium of an isothermal atmosphere if gravity remains constant. 5

(b) A rectangle is immersed vertically in a heavy homogeneous liquid with two of its sides horizontal and at a depth and below the surface. Show that the depth of the centre of pressure is

$$\frac{2}{3}\,\frac{\quad^{2}\qquad\quad^{2}}{\qquad\qquad}$$ 5

(c) If the law connecting the pressure and density of the air is $p \quad ^{n}$, show that the height of the atmosphere is $\frac{n}{n \quad 1}$ times the homogeneous atmosphere neglecting variations of gravity and temperature. 5

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