2021

(July)

MATHEMATICS

(Honours)

(Operations Research)

(HOPT–62:OP2)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Explain the following terms : 2+3+2=7

(i) Convex set

(ii) General linear programming problem

(iii) Feasible and optimum solution to a general LPP

(b) An agriculturist has a farm with 125 acres. He produces radish, mattar and potato. Whatever he raises is fully sold in the market. He gets ₹ 5 for radish per kg, ₹ 4 for mattar per kg and ₹ 5 for potato per kg. The average yield is 1500 kg of radish per acre, 1800 kg of mattar per acre and 1200 kg of potato per acre. To produce each 100 kg of radish and mattar and to produce each 80 kg of potato, a sum of ₹ 12·50 has to be used for manure. Labour required for each acre to raise the crop is 6 man-days for radish and potato each and 5 man-days for mattar. A total of 500 man-days labour at a rate of ₹ 40 per man-day is available. Formulate this as a linear programming problem to maximize the agriculturist's total profit. 8

2. *(a)* A factory, engaged in the manufacturing of pistons, rings and valves for which the profits per unit are ₹ 10, ₹ 6 and ₹ 4 respectively, wants to decide the most profitable mix. It takes one hour of preparatory work, ten hours of machining and two hours of packing and allied formalities for a piston. Corresponding time requirements for rings and valves are 1, 4 and 2, and 1, 5 and 6 hours respectively. The total number of hours available for preparatory work, machining and packing and allied formalities are 100, 600 and 300 respectively. Determine the most profitable mix, assuming that what all produced can be sold. Formulate the LPP. 8

(b) Solve the following LPP graphically : 7

Maximize $Z \quad 2x_1 \quad 4x_2$

subject to the constraints

$$x_1 \quad 2x_2 \quad 5$$
$$x_1 \quad x_2 \quad 4$$
$$x_1, x_2 \quad 0$$

UNIT—II

3. (a) Explain the following terms : 3+3=6

(i) Canonical and standard forms of an LPP

(ii) Slack and surplus variables in a general LPP

(b) Explain what you mean by primal and dual in an LPP. Obtain the dual problem of the following :

Minimize $Z \quad x_1 \quad x_2 \quad x_3$

subject to the constraints

$$x_1 \quad 3x_2 \quad 4x_3 \quad 5$$
$$x_1 \quad 2x_2 \quad 3$$
$$2x_2 \quad x_3 \quad 4$$

$x_1, x_2 \quad 0$ and x_3 is unrestricted in sign.

3+6=9

4. (a) Write down the algorithm of the simplex method to solve an LPP. 9

(b) Rewrite in standard form the following LPP : 6

Maximize $Z \quad 3x_1 \quad 4x_2 \quad 6x_3$

subject to the constraints

$$2x_1 \quad x_2 \quad 2x_3 \quad 6$$
$$3x_1 \quad 2x_2 \quad 8$$
$$7x_1 \quad 3x_2 \quad 5x_3 \quad 9$$

$x_1, x_2 \quad 0$ and x_3 is unrestricted in sign.

UNIT—III

5. *(a)* Solve by simplex method the following LPP : 10

$$\text{Minimize} \quad Z \quad x_1 \quad 3x_2 \quad 2x_3$$

subject to the constraints

$$\begin{array}{rrrl} 3x_1 & x_2 & 2x_3 & 7 \\ & 2x_1 & 4x_2 & 12 \\ 4x_1 & 3x_2 & 8x_3 & 10 \\ & & x_1, x_2, x_3 & 0 \end{array}$$

(b) Describe briefly the following : 3+2=5

(i) Maximin-minimax principle

(ii) Saddle point

6. *(a)* Determine whether the following two-person zero-sum game is strictly determinable and fair. If it so, give the optimum strategies for each player : 5

$$\begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{bmatrix} 0 & 2 \\ -1 & 4 \end{bmatrix} \end{array}$$

(b) For the game with the following pay-off matrix, find the best strategy for each player and the value of a play of the game to *A* and *B* : 5

$$\begin{array}{ccc} & & B \\ & & \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ A & \begin{array}{c} I \\ II \\ III \\ IV \end{array} & \begin{bmatrix} 9 & 3 & 1 & 8 & 0 \\ 6 & 5 & 4 & 6 & 7 \\ 2 & 4 & 3 & 3 & 8 \\ 5 & 6 & 2 & 2 & 1 \end{bmatrix} \end{array}$$

(c) For what value of , the game with the following pay-off matrix is strictly determinable? 5

$$\begin{array}{ccc} & & \textit{Player } B \\ & & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \textit{Player } A & \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} & 6 & 2 \\ -1 & & -7 \\ -2 & 4 & \end{bmatrix} \end{array}$$

UNIT—IV

7. *(a)* Reduce the following game by dominance property and solve it : 9

$$\begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix} \end{array}$$

(b) Solve the following two-person zero-sum games to find the value of the games : 6

$$\begin{array}{cc} & \text{Player } B \\ \text{Player } A & \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} \end{array}$$

8. *(a)* Solve the following game by linear programming technique : 10

$$\begin{array}{ccc} & & \text{Player } Q \\ & & \begin{array}{ccc} Q_1 & Q_2 & Q_3 \end{array} \\ \text{Player } P & \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} & \begin{bmatrix} 9 & 1 & 4 \\ 0 & 6 & 3 \\ 5 & 2 & 8 \end{bmatrix} \end{array}$$

(b) Solve the following 2 3 game graphically : 5

$$\begin{array}{cc} & \text{Player } B \\ \text{Player } A & \begin{bmatrix} 1 & 3 & -3 & 7 \\ 2 & 5 & 4 & -6 \end{bmatrix} \end{array}$$

UNIT—V

9. *(a)* Explain the following terms : 2+2=4

(i) Stochastic matrices and regular stochastic matrices

(ii) Absorbing states

(b) What do you mean by the term 'fixed points of a square matrix'? Find the unique fixed probability vector of the regular stochastic matrix. 2+4=6

$$P \quad \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{matrix}$$

(c) A man either drives his car or takes a train to work each day. Suppose he never takes the train two days in a row; but if he drives to work, then the next day he is just as likely to drive again or take a train. What is the probability that he will change the state from driving to take the train after four days? 5

10. *(a)* Given the transition matrix

$$P \quad \begin{matrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \end{matrix}$$

and the initial probability distribution $p^{(0)} \quad (\frac{2}{3}, 0, \frac{1}{3})$. Find *(i)* $p_{32}^{(2)}$ and $p_{13}^{(2)}$, *(ii)* $p^{(4)}$ and $p_3^{(4)}$, *(iii)* the vector that $p^{(0)}P^n$ approaches and *(iv)* the matrix that $P^{(n)}$ approaches. 1+1½+1½+1=5 *(b)* Let P be the transition matrix of a Markov chain. Then prove that the n-step transition matrix is equal to the nth power of P, i.e., $P^{(n)} \quad P^{n}$. 5

(c) Consider repeated tosses of a fair die. Let X_n be the maximum of the numbers occurring in the first n trails. *(i)* Find the transition matrix p of the Markov chain. *(ii)* Is the matrix regular? *(iii)* Find $p^{(1)}$, the probability distribution after the first toss. *(iv)* Find $p^{(2)}$ and $p^{(3)}$. 5

★ ★ ★