

2021

(July)

STATISTICS

(Honours)

(Survey Sampling and Non-parametric Inference)

[STH-62(TH)]

Marks : 56

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

Answer **five** questions, selecting **one**
from each Unit

UNIT—I

1. (a) What is circular systematic sampling? Write the procedure of drawing such type of samples. How does it differ from linear systematic sampling? 2+2+1=5
- (b) If the population consists of a linear trend, $Y_i = i$ ($i = 1, 2, \dots, k$), then prove that
- $$\text{Var}(\bar{y}_{st}) = \text{Var}(\bar{y}_{sys}) = \text{Var}(\bar{y}_n)_R \quad 7$$

2. (a) Define ratio and regression estimator. Prove that in SRSWOR, for large n , an approximation to the variance of $\hat{R}$ is given by

$$V(\hat{R}) = \frac{1}{n} \frac{f}{\bar{X}^2} \frac{1}{N-1} \sum_{i=1}^N (y_i - R x_i)^2$$

where $f = \frac{n}{N}$ is the sampling fraction. 4+5=9

- (b) Write a short note on the difference estimator of population mean. 3

UNIT—II

3. (a) What is cluster sampling? Write the expression for an unbiased estimate and variance of the estimate of population total for single-stage cluster sampling. 5
- (b) If a simple random sample of n clusters, each containing M elements, is drawn from the N clusters in the population, then show that the sample mean per element $\bar{\bar{y}}$ is an unbiased estimate of $\bar{\bar{Y}}$ with variance

$$V(\bar{\bar{y}}) = \frac{(1-f)}{n} \frac{(NM-1)}{M^2(N-1)} S^2 [1 - (M-1)]$$

$$+ \frac{1}{nM} f S^2 [1 - (M-1)]$$

where ρ is the intra-cluster correlation coefficient. 6

(3)

4. (a) For a single-stage cluster sampling with equal size cluster, under SRSWR, obtain the estimate of population total along with its variance. 7
- (b) Write a note on relative efficiency of cluster sampling with respect to simple random sampling. 4

UNIT—III

5. (a) What is two-stage sampling? Give an example of it. Write the advantages and uses of two-stage sampling. 5
- (b) Show that in two-stage sampling, the sampling mean is an unbiased estimator of population mean. At both the stages, SRSWOR is performed. 6
6. In two-stage sampling, if n units are selected from N primary units and from each selected primary unit, if m units are selected from M secondary units by SRSWOR scheme, then show that sample mean $\bar{y}$ is an unbiased estimator of $\bar{Y}$ and the variance of this estimator is

$$v(\bar{y}) = \frac{1}{n} f S_b^2 + \frac{M}{M} \frac{m}{nm} \frac{S_w^2}{nm} \quad 11$$

(4)

UNIT—IV

7. (a) What do you mean by order statistics? What purpose does it serve? 4
- (b) Obtain the distribution function and hence the p.d.f. of the r th order statistic $X_{(r)}$ in a random sample of size n from a population with continuous distribution function. 7
8. Obtain the expression for joint p.d.f. of two-order statistics and hence obtain the distribution of range. 11

UNIT—V

9. (a) Explain what are meant by non-parametric methods. Distinguish between parametric and non-parametric approaches to the theory of statistical inference. $2+5=7$
- (b) Construct sign test for the location of a univariate population. 4
10. Write notes on the following : $5\frac{1}{2} \times 2 = 11$
- (a) Mann-Whitney test
- (b) Kolmogorov-Smirnov test (for one sample)

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