6/H–29 (viii) (e) (Syllabus–2015)

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(May/June)

MATHEMATICS

(Honours)

(Discrete Mathematics)

(HOPT–62 : OP5)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Prove that $7^n - 2^n$ is divisible by 5 for all $n \in \mathbb{N}$. 5

   (b) If x_1, x_2 and x_3 are non-negative integers, find how many solutions does the equation $x_1 + x_2 + x_3 = 11$ have. 5

   (c) Find the general solution and the unique solutions of the second-order homogeneous recurrence relation

   $$a_n = 5a_{n-1} - 6a_{n-2}$$

   with initial condition $a_0 = 2$, $a_1 = 8$. 5

2. (a) Find which regular polygon has the same number of diagonals as sides. 5

(b) Find the numeric function corresponding to the generating function

$$G(x) = \frac{(1+x)^2}{(1-x)^4}$$ 5

(c) Find the number of rectangles in an $n \times n$ square. 5

UNIT—II

3. (a) Consider the set $\mathbb{Z}$ of integers. Suppose xRy if and only if $x = y^r$ for some positive integer r. Show that R is a partial order on $\mathbb{Z}$. 5

(b) Let D_{30} denotes the poset of all divisors of 30. Show that D_{30} is a lattice. Is it a distributive lattice? Justify. 5

(c) Show that the dual of a distributive lattice is a distributive lattice. 5

4. (a) Draw the Hasse diagram for the positive divisors of 45. 5

(b) Give examples with justification of the following : 2+2=4

(i) Two lattices which are isomorphic

(ii) Two latices which are not isomorphic

(c) Let $(L, \leq)$ be a lattice and $a, b, c \in L$. Show that

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(ii) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$ 3+3=6

UNIT—III

5. (a) Prove the following identities : 2+2=4

(i) $a + (a' \cdot b) = a + b$

(ii) $a' + (a \cdot b) = a' + b$

(b) Construct a logic circuit corresponding to the Boolean function

$$f(x, y, z) = xyz + xy'z$$ 5

(c) Show that a lattice L is modular if and only if for any $a, b, c \in L$, $a \vee [b \wedge (a \vee c)] = (a \vee b) \wedge (a \vee c)$. 6

6. (a) Simplify the following : 5

(i) $(ab' + c)'$

(ii) $(a + b')(a' + b)(a' + b')$

(b) Use Karnaugh map to find a minimal form of the Boolean function $f(x, y, z) = x'y'z' + x'yz' + xyz' + xy'z$. 5

(c) Show that in a Boolean algebra $(x + yz)(y' + x)(y' + z') = x(y' + z')$. 5

UNIT—IV

7. (a) Let the set of vertices be $V = \{1, 2, \cdots, 10\}$. Suppose a vertex u is adjacent to a vertex v if and only if $u + v$ is an even integer. Draw the graph and find the minimum degree δ and the maximum degree Δ. 6

(b) Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. 4

(c) Prove that a connected graph G remains connected after removing an edge e from a circuit. 5

8. (a) Prove that a graph is bipartite if and only if it contains no cycle of odd length. 5

(b) Give examples of the following : 3+3=6

(i) A graph that has a Hamiltonian circuit but no Eulerian circuit

(ii) A graph that has an Eulerian circuit but no Hamiltonian circuit

(c) Show that a graph is K-regular on n-vertices if and only if Kn is even and $n \geq K + 1$. 4

UNIT—V

9. (a) Show that a graph with n-vertices and $\delta \geq \frac{n-1}{2}$ is a connected graph. 4

(b) Let G be a block with $\delta \geq 3$. Show that there is a vertex v such that G-v is also a block. 5

(c) Give examples of the following : 3+3=6

(i) A graph having 2 cut sets

(ii) A graph having edge connectivity and vertex connectivity same

10. (a) Show that the minimum number of vertices separating two non-adjacent vertices u and v is the maximum number of disjoint u-v paths. 6

(b) A tree has $2n$ vertices of degree 1, $3n$ vertices of degree 2 and n vertices of degree 3. Find the number of edges in the tree. 5

(c) State with justification whether the following statements are true or false :

2+2=4

(i) Every graph has a cut vertex

(ii) Every graph has a cut edge

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