6/H–29 (xi) (c) (Syllabus–2019)

2022

(May/June)

MATHEMATICS

(Honours)

(Fluid Mechanics)

(HOP–3)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* The velocity $\vec{q}$ in a three-dimensional flow field for an incompressible fluid is given by

$$\vec{q} = 2x\vec{i} - y\vec{j} - z\vec{k}$$

Is it a possible field? Determine the equations of the streamline passing through the point (1, 1, 1). 5

(b) The velocity components for a two dimensional flow system can be given in the Eulerian system by

$$u = 2x + 2y + 3t$$
$$v = x + y + \tfrac{1}{2}t$$

Find the displacement of a fluid particle in the Lagrangian system. 10

2. (a) Show that

$$u = \frac{2xyz}{(x^2+y^2)^2};\ v = \frac{(x^2-y^2)z}{(x^2+y^2)^2},\ w = \frac{y}{x^2+y^2}$$

are the velocity components of a possible liquid motion. Is this motion irrotational? 7

(b) Show that the variable ellipsoid

$$\frac{x^2}{a^2k^2t^4} + kt^2\left\{\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right\} = 1$$

is a possible form for the boundary surface of a liquid at any time t. 8

UNIT—II

3. A portion of homogenous fluid is confined between two concentric spheres of radii A and a, and is attracted towards their centre

by a force varying inversely as square of distance. The inner spherical surface is suddenly annihilated, and when the radii of the inner and outer surface of the fluid are r and R, the fluid impinges on a solid ball concentric with their surfaces. Prove that the impulsive pressure at any point of the ball for different values of R and r varies as

$$\sqrt{\left\{(a^2 - r^2 - A^2 + R^2)\left[\frac{1}{r} - \frac{1}{R}\right]\right\}}$$ 15

4. Prove that the equation of motion is satisfied for an invisid, incompressible, steady flow with negligible body force whose velocity components are given by

$$q_r = U\left(1 - \frac{A^3}{r^3}\right)\cos\theta$$
$$q_\theta = -U\left(1 + \frac{A^2}{2r^3}\right)\sin\theta,\ q_\phi = 0$$

where A is constant. Find the resultant velocity when $n \to \infty$. 15

UNIT—III

5. (a) Show that $u = 2Axy,\ v = (a^2 + x^2 - y^2)$ are the velocity components of a possible fluid motion. Determine the stream function. 5

(b) Find the lines of flow in the two-dimensional fluid motion given by

$$\phi + i\psi = -\tfrac{1}{2} n(x+iy)^2 e^{2int}$$

Prove that the paths of the particles of the fluid (in polar coordinates) may be obtained by eliminating t from the equations

$$r\cos(nt+\theta) - x_0 = r\sin(nt+\theta) - y_0 = nt(x_0 - y_0)$$ 10

6. Show that the velocity potential

$$\phi = \tfrac{1}{2}\log\frac{(x+a)^2+y^2}{(x-a)^2+y^2}$$

gives a possible motion. Determine the form of streamlines and curves of equal speed. 15

UNIT—IV

7. *(a)* A semi-circular area of radius a is immersed vertically with its diameter horizontal at a depth b. If the circumference be below the centre, prove that the depth of centre of pressure is

$$\frac{3\pi(a^2+4b^2)+32ab}{4(3b\pi+4a)}$$ 10

(b) Find the centre of pressure of a square lamina immersed in a fluid with one vertex in the surface and the diagonal vertical. 5

8. Prove that if the forces per unit mass at (x, y, z) parallel to the axes are $y(a-z)$, $x(a-z)$, xy; then the surfaces of equal pressure are hyperbolic paraboloid and the curves of equal pressure and density are rectangular hyperbolas. 15

UNIT—V

9. *(a)* A hollow cone is placed with its vertex upwards on a horizontal table and a liquid is poured in through a small hole at the vertex, if the cone begins to rise when the weight of the liquid poured in it is equal to its own weight, prove that its weight is to the weight of the liquid to fill the cone as $9 - 3\sqrt{3} : 4$. 9

(b) A right circular cone, closed by a plane base is held with its axis horizontal and is full of water, find the resultant vertical thrust on the lower and upper halves of the curved surface. 6

10. A solid right circular cone of vertical angle 2α is immersed in water with one generator in the surface of the liquid. Prove that the resultant pressure on the curved surface of the cone is to the weight of the fluid displaced by the cone as $\sqrt{1+3\sin^2\alpha} : 1$. Further, show that it is inclined to the axis of the cone of an angle $\cot^{-1}(2\tan\alpha)$ while its inclination to the vertical is given by

$$\tan^{-1}\frac{3\tan\alpha}{1-2\tan^2\alpha}$$ 15

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