6/H-29 (xi) (b) (Syllabus-2019)

2022

(May/June)

MATHEMATICS

(Honours)

(Operation Research)

(HOP-2)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Explain the following terms : 1+2+2=5

(i) Convex set

(ii) General linear programming problem

(iii) Optimum solution to a general linear programming problem

(b) A factory, engaged in the manufacturing of pistons, rings and valves for which the profits per unit are ₹ 10, ₹ 6 and ₹ 4 respectively, wants to decide the most profitable mix. It takes one hour of preparatory work, ten hours of machining and two hours of packing and allied formalities for a piston. Corresponding time requirements for rings and valves are 1, 4 and 2, and 1, 5 and 6 hours respectively. The total number of hours available for preparatory work, machining and packing and allied formalities are 100, 600 and 300 respectively. Determine the most profitable mix, assuming that what all produced can be sold. Formulate the LPP. 5

(c) Solve the following LPP graphically : 5

$$\text{Maximize } Z = x_1 + x_2$$

subject to the constraints

$$x_1 + x_2 \le 1$$
$$-3x_1 + x_2 \ge 3$$
$$x_1 \ge 0,\ x_2 \ge 0$$

2. *(a)* Use simplex method to solve the following LPP : 10

$$\text{Maximize } Z = 2x_1 + 3x_2$$

subject to the constraints

$$x_1 + x_2 \le 4$$
$$-x_1 + x_2 \le 1$$
$$x_1 + 2x_2 \le 5$$
$$x_1 \ge 0,\ x_2 \ge 0$$

(b) Write the dual of the following LPP : 5

$$\text{Maximize } Z = x_1 - x_2 + 3x_3$$

subject to the constraints

$$x_1 + x_2 + x_3 \le 10$$
$$2x_1 - x_2 - x_3 \le 2$$
$$2x_1 - 2x_2 - 3x_3 \le 6$$
$$x_1,\ x_2,\ x_3 \ge 0$$

UNIT—II

3. *(a)* Obtain an initial basic feasible solution to the following transportation problem using the North-West corner rule : 7

	D	*E*	*F*	*G*	*Available*
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

(b) Solve the following transportation problem : 8

Source	Destination 1	2	3	4	Available
1	21	16	25	13	11
2	17	18	14	23	13
3	32	27	18	41	19
Requirement	6	10	12	15	43

4. *(a)* With reference to a transportation problem define the following terms : 2×3=6

(i) Existence of feasible solution

(ii) Basic feasible solution

(iii) Degeneracy

(b) Use Vogel's approximation method to obtain an initial basic feasible solution of the transportation problem : 9

$$\begin{array}{c} \\ A \\ B \\ C \\ \textit{Demand} \end{array} \begin{array}{c} \begin{array}{cccc} D & E & F & G \end{array} \\ \left[\begin{array}{cccc} 11 & 13 & 17 & 14 \\ 16 & 18 & 14 & 10 \\ 21 & 24 & 13 & 10 \end{array}\right] \\ \begin{array}{cccc} 200 & 225 & 275 & 250 \end{array} \end{array} \begin{array}{c} \textit{Available} \\ 250 \\ 300 \\ 400 \\ \\ \end{array}$$

UNIT—III

5. *(a)* Give an Hungarian algorithm to solve an assignment problem. 6

(b) Solve the following assignment problem : 9

	I	*II*	*III*	*IV*	*V*
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	13

6. *(a)* Explain the following terms : 2×3=6

(i) Markov process

(ii) Transition probabilities

(iii) Matrix of transition probabilities

(b) A certain piece of equipment is inspected at the end of each day and classified as just overhauled, good, fair or inoperative. Let us denote the four classifications as states 1, 2, 3, and 4 respectively. If the item is inoperative it is overhauled, a procedure that takes one day. Assume that the working condition of the equipment follows a

Markov process with the following state-transition matrix :

$$\text{Today}\quad\begin{array}{c|cccc} & \multicolumn{4}{c}{\textit{Tomorrow}} \\ & 1 & 2 & 3 & 4 \\ 1 & 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ 2 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 3 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 4 & 1 & 0 & 0 & 0 \end{array}$$

It costs ₹ 125 to overhaul a machine on an average, and ₹ 75 is lost in production, if a machine is found in-operative use the steady state probabilities to compute the expected per day cost of maintenance. 9

UNIT—IV

7. *(a)* Determine whether the following two-person zero-sum game is strictly determinable and fair. If it is so, give the optimum strategy for each player : 5

$$\begin{array}{cc} & \textit{Player B} \\ \textit{Player A} & \begin{bmatrix} -5 & 2 \\ -7 & -4 \end{bmatrix} \end{array}$$

(b) Determine the range of value of p and q that will make the pay-off element a_{22}, a saddle point for the game whose pay-off matrix is given below : 5

$$\begin{array}{cc} & \textit{Player B} \\ \textit{Player A} & \begin{bmatrix} 2 & 4 & 7 \\ 10 & 7 & q \\ 4 & p & 8 \end{bmatrix} \end{array}$$

(c) In a game of matching coins with two players, suppose A wins one unit of value when there are two heads, wins nothing when there are two tails and loses $\frac{1}{2}$ units of value when there are one head and one tail. Determine the pay-off matrix, the best strategies for each player and the value of the game to A. 5

8. *(a)* Describe briefly the following : 3+2=5

(i) Maximin-Minimax principle

(ii) Two-person zero-sum game

(b) For the game with the following pay-off matrix, determine the optimum strategies and the value of the game : 5

$$\begin{array}{cc} & P_2 \\ P_1 & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

(c) For what value of λ, the game with the following pay-off matrix is strictly determinable? 5

Player B

$$\text{Player } A \quad \begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & \lambda & 6 & 2 \\ A_2 & -1 & \lambda & -7 \\ A_3 & -2 & 4 & \lambda \end{array}$$

UNIT—V

9. (a) Solve the following 3×2 games graphically : 5

B's Strategy

$$\text{A's Strategy} \quad \begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & 3 & -4 \\ A_2 & 2 & 5 \\ A_3 & -2 & 8 \end{array}$$

(b) For the following pay-off table, transform the zero-sum game into an equivalent linear programming problem and solve it by simplex method : 10

Player Q

$$\text{Player } P \quad \begin{array}{c|ccc} & Q_1 & Q_2 & Q_3 \\ \hline P_1 & 9 & 1 & 4 \\ P_2 & 0 & 6 & 3 \\ P_3 & 5 & 2 & 8 \end{array}$$

10. (a) Reduce the following game by dominance property and solve it : 8

Player B

Player A	I	II	III	IV
I	3	2	4	0
II	3	4	2	4
III	4	2	4	0
IV	0	4	0	8

(b) Consider the following 2×2 game :

$$\begin{pmatrix} 4 & 7 \\ 6 & 5 \end{pmatrix}$$

(i) Does it have a saddle point?

(ii) Is it correct to state that the value of game G, will satisfy $5 < G < 6$?

(iii) Determine the frequency of optimum strategies by matrix oddment method and find the value of game.

1+1+5=7

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