6/H-29 (viii) (b) (Syllabus-2015)

2022

(May/June)

MATHEMATICS

(Honours)

(Operations Research)

(HOPT-62 : OP2)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* A manufacturing company produces two types of computer monitors—colour and monochrome. The data in the manufacturing context are—

   (i) 6 days are required to complete one unit of each type of monitor and only 240 working days are available in a year;

   (ii) the profit for a colour monitor is ₹ 1,500 and that for a monochrome is ₹ 1,000;

(iii) the marketing department reports that only 20 units of the colour and 30 units of monochrome monitors can be sold in a year.

Formulate the linear programming model to determine how many units of each type of monitors should be manufactured so that the total profit is maximized. 6

(b) A firm uses lathes, milling machines and grinding machines to produce two machine parts. The following table represents the machining time required for each part, the maximum machining time available on the different machines and the profit on each machine part :

Type of machine used	*Machining time required for the machine part* (in minute)		*Maximum time available per week* (in minute)
	Part—I	*Part—II*	
Lathes	12	6	3000
Milling machines	4	10	2000
Grinding machines	2	3	900
Profit per unit	₹ 40	₹ 100	

Find the number of parts I and II to be manufactured per week to maximize the profit. 9

2. *(a)* Consider two different types of foodstuffs say F_1 and F_2. Assume that these foodstuffs contain vitamins V_1, V_2 and V_3 respectively. Minimum daily requirements of these vitamins are 1 mg of V_1, 50 mg of V_2 and 10 mg of V_3. Suppose that the foodstuff F_1 contains 1 mg of V_1, 100 mg of V_2 and 10 mg of V_3 whereas foodstuff F_2 contains 1 mg of V_1, 10 mg of V_2 and 100 mg of V_3. Cost of one unit of foodstuff F_1 is ₹ 1 and that of F_2 is ₹ 1·5. Find, by graphical method, the minimum cost of the diet that would supply the body at least the minimum requirement of each vitamin. 8

(b) Solve the following LPP graphically : 7

Maximize $Z = 3x + 2y$

subject to the constraints

$$-2x + 3y \leq 9$$
$$3x - 2y \leq -20$$
$$x \geq 0,\ y \geq 0$$

UNIT—II

3. *(a)* Explain the following terms : 2+2=4

(i) Slack and surplus variables in a general LPP

(ii) Canonical form of an LPP

(b) Obtain the dual of the following primal problem : 6

Minimize $Z = 3x_1 - 2x_2 + x_3$

subject to

$$2x_1 - 3x_2 + x_3 \le 5$$
$$4x_1 - 2x_2 \ge 9$$
$$-8x_1 + 4x_2 + 3x_3 = 8$$

$x_1 \ge 0$, $x_2 \ge 0$ and x_3 is unrestricted.

(c) Express the following LPP in the standard form : 5

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

subject to

$$2x_1 - 3x_2 \le 3$$
$$x_1 + 2x_2 + 3x_3 \ge 5$$
$$3x_1 + 2x_3 \le 2$$

$x_1 \ge 0$, $x_2 \ge 0$ and x_3 is unrestricted.

4. (a) Write down the algorithm of the simplex method to solve a linear programming problem. 9

(b) Construct the initial simplex table for the following LPP : 6

Maximize $Z = 4x_1 + 10x_2$

subject to the constraints

$$2x_1 + x_2 \le 50$$
$$2x_1 + 5x_2 \le 100$$
$$2x_1 + 3x_2 \le 90$$
$$x_1 \ge 0,\ x_2 \ge 0$$

UNIT—III

5. (a) Solve the following problem by simplex method : 9

Maximize $Z = 3x_1 - x_2$

subject to

$$2x_1 + x_2 \ge 2$$
$$x_1 + 3x_2 \le 3$$
$$x_2 \le 4$$
$$x_1, x_2 \ge 0$$

(b) Explain the following terms giving suitable examples : 2×3=6

(i) Competitive games

(ii) Mixed strategy

(iii) Saddle point

6. (a) In a rectangular game, the pay-off matrix is given by

$$\begin{array}{cc} & \textit{Player B} \\ \textit{Player A} & \begin{bmatrix} 10 & 5 & 5 & 20 & 4 \\ 11 & 15 & 10 & 17 & 25 \\ 7 & 12 & 8 & 9 & 8 \\ 5 & 13 & 9 & 10 & 5 \end{bmatrix} \end{array}$$

Determine the best strategies for *Player A* and *Player B*. State, giving reasons, whether the players will use pure or mixed strategies. What is the value of the game? Is the game (i) fair and (ii) strictly determinable? 8

(b) For what value of p, the game with the following pay-off matrix is strictly determinable? 5

$$\begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{bmatrix} p & 7 & 3 \\ -2 & p & -8 \\ -3 & 4 & p \end{bmatrix} \end{array}$$

(c) Consider the game :

$$\begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{bmatrix} 4 & 1 \\ 3 & a \end{bmatrix} \end{array}$$

Find a solution to the game, if $1 < a < 3$. 2

UNIT—IV

7. (a) Solve the following game : 5

$$\begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{array}$$

(b) Solve the game whose pay-off matrix is given by

$$\begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{bmatrix} 6 & -2 & 1 \\ 9 & 15 & 2 \\ 3 & -1 & 4 \\ 7 & 13 & 0 \end{bmatrix} \end{array}$$

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8. (a) Solve the following game graphically : 6

$$\begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{bmatrix} 1 & 0 & 4 & -1 \\ -1 & 1 & -2 & 5 \end{bmatrix} \end{array}$$

(b) Solve the following game by linear programming technique : 9

$$\begin{array}{cc} & \textit{Player } B \\ \textit{Player } A & \begin{bmatrix} 2 & -3 & 4 \\ -3 & 4 & -5 \\ 4 & -5 & 6 \end{bmatrix} \end{array}$$

UNIT—V

9. (a) Describe briefly the following : 2×3=6

(i) Fixed points of square matrices

(ii) Markov chain

(iii) Absorbing state

(b) A salesman's territory consists of three cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in the city B. However, if he sells in either B or C, then the next day he is twice as likely to sell in city A as in the other city.

(i) Find the transition matrix.

(ii) What is the probability that the salesman will shift from city B to city C after 3 days?

(iii) In the long run, how often does he sell in each of the cities? 9

10. *(a)* Let

$$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

be a stochastic matrix and let $u = (u_1, u_2, u_3)$ be a probability vector. Show that uA is also a probability vector. 5

(b) On January 1st (this year), bakery A had 40% of its local market while the other two bakeries B and C had 40% and 20% respectively of the market. Based upon a study by a marketing research firm, the following facts were compiled : Bakery A retains 90% of its customers while gaining 5% of B's customers and 10% of C's customers. Bakery B retains 85% of its customers while gaining 5% of A's customers and

7% of C's customers. Bakery C retains 83% of its customers and gains 5% of A's customers and 10% of B's customers.

(i) Find the transition matrix P.

(ii) What will each firm's share be on January 1st next year?

(iii) What will each firm's market share be in the long run? 10

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