6/H-29 (xi) (f) (Syllabus-2019)

2022

(May/June)

MATHEMATICS

(Honours)

(Probability Theory)

(HOP-6)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each unit

UNIT—I

1. *(a)* Define the terms 'random experiment' and sample space with examples. 2+2=4

   (b) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box, at random. Find the probability that among the balls drawn there is at least one ball of each colour. 6

   (c) State and prove the theorem of total probability. 5

2. (a) State and prove Bayes' theorem. 2+4=6

(b) The chances of X, Y, Z becoming managers of a certain company are 4 : 2 : 3. The probabilities that bonus scheme will be introduced if X, Y, Z become managers are 0·3. 0·5, 0·8 respectively. If the bonus scheme was introduced, what is the probability that X would be the manager? 5

(c) If A, B and C are mutually independent events, then show that $A \cup B$ and C are independent. 4

UNIT—II

3. (a) If F is the distribution function of the random variable X and if $a < b$, then show that $P(a < X \le b) = F(b) - F(a)$. Hence, show that

$$P(a \le X \le b) = P(X = a) + F(b) - F(a)$$

and $P(a \le X < b) = P(a < X < b) + P(X = a)$

3+1½+1½=6

(b) A discrete random variable X has the following probability function :

X, x	:	0	1	2	3	4	5	6	7
$p(x)$	:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2 + K$

(i) Find K.

(ii) Evaluate $P(X < 6)$, $P(X \ge 6)$, $P(0 < X < 5)$

(iii) Determine the distribution function of X. 3+3+3=9

4. (a) Let X be a random variable such that

$$P(X = -2) = P(X = -1),\ P(X = 2) = P(X = 1)$$

and $P(X > 0) = P(X < 0) = P(X = 0)$.

Obtain the probability mass function of X and its distribution function. 3+2=5

(b) Define the probability density function of a continuous random variable X, with an example. The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} Kx^2(1-x)\,; & 0 < x < 1 \\ 0\,; & \text{elsewhere} \end{cases}$$

Find the constant K. 2½+2½=5

(c) Joint distribution of X and Y is given by $f(x, y) = 4xye^{-(x^2+y^2)}\,; x \ge 0,\ y \ge 0$. Test whether x and y are independent. For the above joint distribution, find the conditional density of X given $Y = y$. 3+2=5

UNIT—III

5. (a) If X and Y are discrete random variables, show that
$$E(X+Y) = E(X) + E(Y)$$ 7

(b) If the pdf of a random variable X is given by $f(x) = Ce^{-(x^2+2x+3)}; -\infty < x < \infty$, where C is a constant, find the expectation and variance of X. 4+4=8

6. (a) Let a random variable X assumes value r with the probability law

$$P(X = r) = q^{r-1}p; r = 1, 2, 3, \cdots \text{ and } q + p = 1$$

Find the moment generating function of X and hence its mean and variance. 3+1+2=6

(b) (i) Define the characteristic function of a real variable t for discrete and continuous probability distributions. 2+2=4

(ii) Find the characteristic function of the distribution given by $dF(x) = \frac{1}{2}e^{-|x|}\,dx$. 5

UNIT—IV

7. (a) Show that hypergeometric distribution tends to binomial distribution, under limiting conditions. 5

(b) Find the first four moments of Poisson distribution about origin. 5

(c) A dice is thrown 3600 times. Show that the probability that the number of sixes lies between 550 and 650 is at least $\frac{4}{5}$. 5

8. (a) If X is normally distributed and its mean is 12 and variance is 16, find the probability of—

(i) $x \geq 20$;

(ii) $x \leq 20$;

(iii) $0 \leq X \leq 12$.

Given that $P(0 \leq Z \leq 2) = 0 \cdot 4772$ and $P(0 \leq Z \leq 3) = 0 \cdot 4987$ where Z is a standard normal variate. 2+2+2=6

(b) After correcting 50 pages of the proof of a book, the proofreader finds that there are, on the average, 2 errors per 5 pages. How many pages would one expect to find with 0, 1, 2, 3, and 4 errors in 1000 pages of the first print of the book? 5

(c) If X and Y are independent normal variates with means 6, 7 and variances 9, 16 respectively, find λ such that

$$P(2X+Y\leq\lambda)=P(4X-3Y\geq 4\lambda)$$ 4

UNIT—V

9. (a) Derive Chi-square distribution using method of moment generating function. 5

(b) If X is a chi-square variate with n degrees of freedom, prove that for large n, $\sqrt{2X}\sim N(\sqrt{2n},1)$. 5

(c) The heights of six randomly chosen sailors are in inches :

63, 65, 68, 69, 71 and 72

Those of ten randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73. Discuss the light that these data throw on the suggestion that sailors are on the average taller than the soldiers. 5

10. (a) Pumpkins were grown under two experimental conditions. Two random samples of 11 and 9 pumpkins show the sample standard deviations of their weights as 0·8 and 0·5 respectively. Assuming that the weight distributions

are normal, test the hypothesis that the true variances are equal, against the alternative that they are not at 10% level.
[Assume that $P(F_{10,8}\geq 3\cdot 35)=0\cdot 05$ and $P(F_{8,10}\geq 3\cdot 07)=0\cdot 05$] 7

(b) State the central limit theorem. 2

(c) Obtain the sampling distribution of sample mean and variance from an independent normal distribution. 3+3=6

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