6/H–28 (vii) (Syllabus–2015)

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(May/June)

STATISTICS

(Honours)

(Statistical Inference)

[STEH–61 (TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define minimum variance unbiased (MVU) estimator. Show that an MVU estimator is unique. 1+5=6

(b) Let $\{T_n\}$ be a sequence of estimators such that for all $\theta \in \Theta$

(i) $E(T_n) \to \gamma(\theta)$, $n \to \infty$

(ii) $\text{Var}(T_n) \to 0$, as $n \to \infty$

Show that T_n is a consistent estimator of $\gamma(\theta)$. 6

2. (a) Prove that under certain regularity conditions to be stated by you, the variance of an unbiased estimator T for $\gamma(\theta)$, satisfies the inequality 8

$$\text{var}(T) \geq \frac{[\gamma'(\theta)]^2}{E\left[\frac{\partial}{\partial\theta}[\log f(X_1, X_2, \cdots, X_n)]\right]^2}$$

(b) Let $x_1, x_2, \cdots, x_n$ be a random sample from a population with p.d.f. :

$$f(x, \theta) = \theta x^{\theta-1};\ 0 < x < 1,\ \theta > 0$$

Show $t = \prod_{i=1}^{n} x_i$ is sufficient for θ. 4

UNIT—II

3. (a) Explain the principle of maximum likelihood (ML) for estimation of population parameter. 4

(b) Prove that the maximum likelihood estimator of the parameter θ of a population having density function

$$f(x; \theta) = \frac{2}{\theta^2}(\theta - x);\ 0 < x < \theta$$

for a sample of unit size is $2x$, x being the sample value. Show also that the estimator is biased. 4

(c) For the distribution with density function

$$f(x, \theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}};\ x > 0,\ \theta > 0$$

Obtain the estimator of θ by the method of moments. 3

4. (a) Discuss the concept of interval estimation and explain the difference between point and interval estimations. 2+4=6

(b) Obtain a large sample $100(1-\alpha)\%$ confidence interval for the parameter θ in random sampling from the population with density function 5

$$f(x, \theta) = \theta e^{-\theta x};\ x > 0$$

UNIT—III

5. (a) Define the following terms : 2+1+1+1=5

(i) Errors of first and second kinds

(ii) The best critical region

(iii) Power function of a test

(iv) Level of significance

(b) Let p be the probability that a coin will fall head in a single toss, in order to test $H_0: p = \frac{1}{2}$ again $H_1: p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtain. Find the probability of type–I error and power of the test. 6

6. (a) Explain the terms : 2×3=6

(i) Most powerful test

(ii) Uniformly most powerful test

(iii) Unbiased test

(b) Let X have a p.d.f. of the form

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}; & x > 0, \theta > 0 \\ 0\ ; & \text{otherwise} \end{cases}$$

To test $H_0: \theta = 2$ against $H_1: \theta = 1$, use a random sample X_1, X_2 of size 2 and define a critical region

$$C = \{(x_1, x_2) : 9 \cdot 5 \leq x_1 + x_2\}$$

Find—

(i) power function of the test;

(ii) significance level of the test. 5

UNIT—IV

7. (a) State Neyman-Pearson lemma. 1

(b) Prove that if W is a most powerful (MP) region for testing $H_0: \theta = \theta_0$ against $\theta = \theta_1$, then it is necessarily unbiased. 5

(c) Use the Neyman-Pearson lemma to obtain the region for testing $\theta = \theta_0$ against $\theta = \theta_1 > \theta_0$ and $\theta = \theta_1 < \theta_0$, in the case of a normal population $N(\theta, \sigma^2)$, where σ^2 is known. 5

8. Describe Wald's SPRT. Develop the SPRT for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (> \theta_0)$ based on a random sample from a normal population with parameters (θ, σ^2). Obtain its OC and ASN functions. 4+7=11

UNIT—V

9. (a) What do you mean by large sample tests? State central limit theorem and write its applications. 2+1+2=5

(b) Obtain the large sample test for difference of proportions. 6

10. (a) Derive the expression for the standard error of a mean of a random sample size n from a population with variance σ^2. 3

(b) Obtain the test of significance for difference of two population means. 5

(c) The means of two single large samples of 1000 and 2000 members are 67·5 inches and 68·0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2·5 inches? (Test at 5% level of significance) 3

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