6/H—28 (vii) (Syllabus-2015)

2019

(April)

STATISTICS

(Honours)

(**Statistical Inference**)

[STEH-61(TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* What are the criteria for a good estimator? Explain each in brief. 6

   (b) Define uniformly minimum variance unbiased estimator (UMVUE). Let T_0 be an UMVUE, while T_1 be an unbiased estimator with efficiency $e_0 < 1$. Prove that no unbiased linear combination of T_0 and T_1 can be UMVUE. 1+5=6

2. *(a)* Define consistent estimator and state the sufficient conditions for consistency. 2

(b) Let $X_1, X_2, \cdots, X_n$ denote a random sample from a Poisson distribution with parameter λ. Examine whether the unbiased estimator of λ, $\overline{X}$ attains the Cramer-Rao lower bound. 6

(c) Let $X_1, X_2, \cdots, X_n$ denote a random sample from the distribution that has pmf

$$f(x; \theta) = \theta^x (1-\theta)^{1-x}, \ x = 0, 1; \ 0 < \theta < 1$$

Show that

$$\sum_{i=1}^{n} X_i$$

is sufficient for θ. 4

UNIT—II

3. *(a)* Name at least five methods of point estimation. Explain in detail the method of moments for estimating the parameter of a population. 1+5=6

(b) Estimate α and β from the following distribution by the method of moments : 5

$$f(x; \alpha, \beta) = \frac{\beta^\alpha}{\sqrt{\alpha}} x^{\alpha - 1} e^{-\beta x}, \ x > 0$$

(Continued)

4. *(a)* Discuss the concept of interval estimation and explain the difference between point estimation and interval estimation. 4+2=6

(b) Find the 95% confidence limits for λ of Poisson distribution with pmf

$$f(x; \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}; \ x = 0, 1, 2, \cdots$$ 5

UNIT—III

5. *(a)* Define the terms : 5

(i) Level of significance

(ii) Size of a test

(iii) Power of a test

(iv) Type–I and Type–II errors

(b) In order to test whether a coin is perfect, the coin is tossed 4 times. The null hypothesis of perfectness is rejected if more than 4 heads are obtained. What is the Type–I error? Find the probability of Type–II error when the corresponding probability of a head is 0·2. What is the power of the test? 2+2+2=6

6. *(a)* Explain the terms : 2+2+2=6

(i) Most powerful test

(ii) Uniformly most powerful test

(iii) Unbiased test

(Turn Over)

(b) Given an observation x from uniform $(0, \theta)$ distribution with density function

$$f(x, \theta) = \begin{cases} 1/\theta & , \ 0 \le x \le \theta \\ 0 & , \ \text{otherwise} \end{cases}$$

To test $H_0 : \theta = 1 \cdot 5$ against $H_1 : \theta = 2 \cdot 5$ with critical region $x \ge 1$, compute size of the test and power of the test. 5

UNIT—IV

7. (a) State and prove Neyman-Pearson lemma for testing a simple hypothesis against a simple alternative. 1+5=6

(b) Construct the likelihood ratio test for testing $H_0 : \theta = \theta_0$ against all its alternatives in $N(\theta, \sigma^2)$, where σ^2 is known. 5

8. (a) Give the SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ in the sampling from a normal distribution. 6

(b) Given a random sample $X_1, X_2, \cdots, X_n$ from the distribution with p.d.f.

$$f(x, \theta) = \theta e^{-\theta x}, \ x > 0$$

Obtain the best critical region for testing $H_0 : \theta = \theta_0$, against $H_1 : \theta = \theta_1$ $(\theta_1 < \theta_0)$. 5

(Continued)

UNIT—V

9. (a) What do you mean by large sample tests? State central limit theorem and write its applications. 2+1+2=5

(b) Obtain the large sample test for difference of two binomial proportions. 6

10. (a) Derive the expression for the standard error of the sample mean from a sample of size n. 3

(b) Obtain the test of significance for difference between two population means. 5

(c) Obtain the test of significance for single mean from normal population with mean μ and variance σ^2. 3

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6/H—28 (viii) (Syllabus-2015)

2019

(April)

STATISTICS

(Honours)

(Survey Sampling and Non-parametric Inference)

[STEH-62(TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. (a) Describe linear systematic sampling. How does it differ from circular systematic sampling? Write the advantages and disadvantages of systematic sampling. 3+3+3=9

(b) Show that the variance of the mean of a systematic sample is

$$V(\bar{y}_{sys}) = \frac{N-1}{N} S^2 - \frac{k(n-1)}{N} S^2_{wsy}$$

where

$$S^2_{wsy} = \frac{1}{k(n-1)} \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y}_i)^2$$ 3

2. Write short notes on the following : 3×4=12

(a) Interpenetrating sub-samples (IPSS)

(b) Difference estimation

(c) Ratio method of estimation

(d) Regression method of estimation

UNIT—II

3. (a) Write briefly, with example, how you would draw a cluster sample. 3

(b) If n clusters are selected by SRSWR scheme from N clusters, where ith cluster is of size M_i, then show that $\bar{y}_{cl}$ is an unbiased estimator of $\bar{Y}$ and the variance of this estimator is

$$V(\bar{y}_{cl}) = \frac{1-f}{n} S^2_{bc}$$

where

$$S^2_{bc} = \frac{1}{N-1} \sum \left(\frac{M_i}{\bar{M}} \bar{Y}_i - \bar{Y} \right)^2$$

and $f = n/N$. 3+5=8

4. (a) Define intra-class correlation coefficient. Write the expression for intra-class correlation coefficient for single-stage cluster sampling. 5

(b) Obtain the expression for variance of the sample mean in terms of intra-class correlation coefficient. 6

UNIT—III

5. (a) With an example, write briefly the method of drawing two-stage sampling. Write the advantage and use of two-stage sampling. 6

(b) Show that in two-stage sampling, the sample mean is an unbiased estimator of population mean. SRSWOR is performed at both the stages. 5

6. (a) In two-stage sampling, if n first stage units are selected and from each selected first stage unit m, second stage units are selected under SRSWOR scheme, then show that the estimator of variance of sample mean $\bar{y}$ is given by

$$v(\bar{y}) = \frac{1-f}{n} s_b^2 + \frac{f(1-f_1)}{nm} s_w^2$$

where

$$f = \frac{n}{m}, f_1 \frac{m}{M}, s_b^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\bar{y}_i - \bar{y})^2$$

and

$$s_w^2 = \frac{1}{n(m-1)}\sum_{i=1}\sum_{j=1}^{m}(y_{ij} - \bar{y}_i)^2$$ 9

(b) Write the expression for the variances of the sample mean if n clusters are selected under SRSWR and if m elements are selected from each selected cluster under SRSWOR. 2

UNIT—IV

7. (a) Write some advantages and applications of order statistics. 4

(b) Obtain the distribution of r-th order statistics from uniform distribution. 7

8. (a) Show that in odd samples of size n from $U[0, 1]$ population, the mean and variance of the distribution of median are $\frac{1}{2}$ and $1/[4(n+2)]$ respectively. 6

(b) In a random sample of size n from uniform $U[0, 1]$ population, obtain the p.d.f. of $W_{rs} = X_{(s)} - X_{(r)}$ and identify its distribution. 5

UNIT—V

9. (a) What are the different assumptions associated with non-parametric statistical methods? Write some areas of applications. 4

(b) Write the advantages of non-parametric over parametric methods. 4

(c) Construct sign test for the location of a univariate population. 3

10. Write short notes on the following : 5½×2=11

(a) Wilcoxon rank test

(b) Median test

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