6/H—28 (vii) (Syllabus–2015)

2 0 1 8

(April)

STATISTICS

(Honours)

(Statistical Inference)

[STEH–61(TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Define MVUE. Show that if T_1 is an MVUE of $\gamma(\theta)$ and T_2 is any other unbiased estimator of $\gamma(\theta)$ with efficiency $e < 1$, then no unbiased linear combination of T_1 and T_2 can be an MVUE of $\gamma(\theta)$.

2+4=6

(b) Define consistency of an estimator. Show that the proportion of successes in a series of n trials with constant probability of success p for each trial, is a consistent estimator of population proportion of success P. 2+4

2. (a) State and prove Cramer-Rao inequality. 1+5

(b) For a random sample x_i $(i = 1, 2, \cdots, n)$ from an exponential distribution with p.d.f.

$$f(x, \theta) = \frac{1}{\theta}\exp\left[-\frac{x}{\theta}\right], \; x > 0, \theta > 0$$

obtain an unbiased and sufficient estimator for θ. 2+4

UNIT—II

3. (a) Define maximum likelihood estimator and state its properties. Find the maximum likelihood estimator of the parameter μ of $N(\mu, \sigma^2)$, when σ^2 is known. 1+2+3

(b) Obtain the maximum likelihood estimator for the distribution having the probability mass function

$$f(x, \theta) = \theta^x(1-\theta)^{x-1}, \; x = 0, 1, 2, \cdots$$

$$0 \le \theta \le 1$$

(Continued

4. (a) Explain the general method of constructing confidence interval for parameter of a population. 5

(b) Construct the confidence interval for mean parameter μ of normal population with known σ^2 and proportion parameter p of binomial population with known n. 6

UNIT—III

5. (a) Explain what is meant by a statistical hypothesis. Also discuss the two types of error that arise in testing of hypothesis. 2+3=5

(b) If $x \ge 1$ is the critical region for testing $\theta = 2$ against the alternative $\theta = 1$, on the basis of a single observation from the population

$$f(x, \theta) = \frac{1}{\theta}e^{-x/\theta}; \; 0 < x < \infty$$

evaluate the type-I, type-II errors and the power function of the test. 2+2+2=6

6. (a) Explain the terms 'most powerful test', 'uniformly most powerful test' and 'unbiased test'. 2+2+2=6

(b) Let $X_1, X_2, X_3, \cdots, X_n$ be i.i.d. $N(\mu, \sigma^2)$ random variables, where σ^2 is known. Find the MP test to test the null hypothesis $H_0 : \mu = \mu_0$ against the alternative $H_1 : \mu = \mu_1$. 5

(Turn Over)

UNIT—IV

7. (a) State Neyman-Pearson lemma. What are its differences from likelihood ratio test? 2+3=5

(b) Construct the likelihood ratio test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ based on a sample of size n from $N(\mu, \sigma^2)$, $\sigma^2 > 0$. 6

8. (a) Define OC function and ASN function of SPRT. 2+2=4

(b) Give the SPRT for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$ in the sampling from a normal density

$$f(x, \theta) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2} ; -\infty < x < \infty$$

where σ is known. Also obtain its OC function. 3+4=7

UNIT—V

9. (a) Differentiate between large sample and small sample tests and discuss their consequences in testing of hypothesis problems. How does the central limit theorem help in deriving large sample tests? 2+2+2=6

(Continued)

(b) Obtain the large sample test for single binomial proportion. 5

10. (a) Describe large sample test of significance for single proportion. Also write down the confidence interval for the proportion. 6

(b) Obtain the test procedure (for large samples) for the test of significance for difference of means. 5

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