Elementary Number Theory & Advanced Algebra

(Number of Teaching hours: 80; Time:3 hrs; Marks: 100)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I: Divisibility in the set of integers; basic properties; the division algorithm; gcd; elementary properties; the Euclidean algorithm; lcm; primes (in the set of natural numbers); fundamental theorem of arithmetic; Euclid's proof of the infinitude of primes; arbitrary gaps in the distribution of primes; Congruences in the set of integers modulo a positive integer; basic properties; complete residue system; reduced residue system; Euler's φ - function; Fermat's theorem: Euler's generalization of Fermat's theorem; applications, Wilson's theorem.

UNIT II: Solution of congruences; linear congruences; Chinese remainder theorem; congruences of higher degree modulo a prime.
Some functions of Number Theory-- Greatest integer function; elementary properties; Arithmetic functions; multiplicative functions; functions such as $\varphi(n)$, $\mu(n)$, $\tau(n)$, $\sigma(n)$, $\sigma_k(n)$;

UNIT III: Normal subgroups, examples; conditions for a subgroup to be normal; center of a group; examples; quotient group; homomorphism, kernel and image of homomorphism, isomorphism of groups – examples and elementary properties. Fundamental theorem of group homomorphism; isomorphism theorems; automorphisms; inner automorphisms; examples; rings (motivation through **Z**) : definitions and examples of (i) rings with identity, (ii) commutative rings, (iii) rings with and without zero-divisors, integral domains, (iv) division rings (v) fields (examples to include $\mathbf{Z}_p$, integers mod p , fields Q, R, C, polynomial rings R[x], matrix rings $M_n(R)$); basic properties of rings; characteristic of rings; finite integral domains; ($\mathbf{Z}_p$ as an example);
Subrings; ideals: right, left and two-sided; generated by a subset, more specifically by a finite number of elements in a commutative ring with 1; principal ideals; examples of ideals in **Z**, $\mathbf{Z}_n$ $M_n(R)$; prime ideals, maximal ideals in a commutative ring with 1; examples; quotient ring, $\mathbf{Z}_n$ as a quotient ring.

UNIT IV: Principal ideals; examples of ideals in **Z**, $\mathbf{Z}_n$ $M_n(R)$; prime ideals, maximal ideals in a commutative ring with 1; examples; quotient ring, $\mathbf{Z}_n$ as a quotient ring. Ring homomorphisms; kernels; isomorphism; homomorphisms and isomorphism theorems including the correspondence theorem; determination of ideals in $\mathbf{Z}_p$; divisibility in integral domains (with 1); units, associates, prime elements, irreducible elements, gcd, Euclidean domain, principal ideal domain, unique factorisation domains – definition, examples and basic results

UNIT V: Vector spaces (motivation through R^2, R^3) – examples, basic properties; subspaces; homomorphisms or linear maps between vector spaces; isomorphisms; standard homomorphism and isomorphism theorems; direct sum (internal and external): linear dependence and independence; basis, dimension; vector space axioms for the set L(V, W) of linear maps from V to W; rank and nullity of a linear transformation; "rank + nullity = dimension" theorem. Matrix- representation of linear transformations; similar matrices, change of basis theorem (without proof, statement only and its application); ; equality of rank of a linear transformation and rank of the associated matrix.

Books

Text Books:

1. Niven, I., Zuckerman, H.S., and Montgomery, H. L. : An introduction to the Theory of Numbers, Wiley Eastern Ltd., 2000 Edition.
2. Burton, David M. : Elementary Number Theory, Universal Book Stall, 2001 Edition.
3. Herstein, I. N.: Topics in Algebra, Vikas Pub. House, 1988 Edition (reprint 1998).
4. Fraleigh, J. B. : A First Course in Abstract Algebra, Narosa Publishing House, 1999 Edition.
5. Saikia, P. K.: Linear Algebra, Pearson, Delhi, 2009 Edition.

Reference Book:

1. Bhattacharya, P.B., Jain, S. K., and Nagpaul, S. R.: A First Course in Linear Algebra, Wiley Eastern Publication, 2001 Edition.

3. Hoffman, K. and Kunze, R.: Linear Algebra, second edition, PHI Learning Pvt Ltd, New Delhi, 1971 Edition (reprint 1996).

4. Telang, S. G. : Number Theory, Tata McGraw-Hill, New Delhi, 1996 Edition.

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Differential Equations & Advanced Dynamics .

(100 marks, 80 lectures)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I: Linear equations of second and third order with constant coefficients – complementary functons and particular integrals for $x^n e^{ax}$, $e^{ax}sin(mx)$, $e^{ax}cos(mx)$, $x^n sin(mx)$, $x^n cos(mx)$; equations of type $a_1 x^2 y'' + a_2 x y' + a_3 y = f(x)$;
Linear differential equations of second order with variable coefficients; homogeneous equations; exact equations; transformation of the equation by changing the dependent variable/the independent variable, Normal form
Method of variation of parameters; simultaneous equations; total differential equation Pdx + Qdy + Rdz = 0

UNIT II: Partial differential equation. Formation of equation, solutions of linear equations of first order, Lagrange's methods, Non linear partial differential equations of first order- Standard forms I, II, III & IV. Integral surfaces passing through a given curve, orthogonal surfaces, non-linear equations of first order, Charpit's method.

(Introduction to the following concepts should be made in vector as well as Cartesian method)

UNIT III: Motion on a rough curve; the cycloid and its dynamical properties: cycloidal motion with resistance; Central forces, Central orbit; Centre of force; motion of a particle under a central force; description of a central conic under a central force; use of reciprocal polar coordinates; stability of a nearly circular orbit.
Use of pedal coordinates and pedal equations; apse; apsidal distance; apsidal angle; perihelion and aphelion; Kepler's laws of planetary motionand its deductions; a more accurate form of the third law.

UNIT IV: Moments and products of inertia; uniform rod, a rectangular lamina, a parallelepiped, a circular ring and disc; theorems of parallel and perpendicular axes about a fixed axis; principal axes(only definition), vector angular velocity of a rigid body; vector angular momentum of a rigid body about a fixed point, principal axes; kinetic energy of a rigid body rotating about a fixed point; momental ellipsoid; equimomental systems; coplanar distributions; general motion of a rigid body.

UNIT V: Motion of a rigid body in two-dimensions; Problems illustrating the laws of motion, mtion of a uniform solid circular cylinder down a rough inclined plane; motion of a circular hoop on a rough inclined plane; laws of conservation of angular momentum; problems illustrating the laws of conservation of angular momentum.
The law of conservation of energy; problems illustrating the law of conservation of energy; Impulse of a force; problems illustrating impulsive.

Books

Text Books:

1. Raisinghania, M.D.: Ordinary and Partial Differential Equations, S. Chand & Co. Ltd., New Delhi, 2002 Edition.
2. Piaggio, I.: An Elementary Treatise on Differential Equations and Applications, G. Bell & Sons, 2000 Edition.
3. Sneddon, I. N.: Elements of Partial Differential Equation, McGraw Hill. International Edition 1957.
4. Chorlton, F.: Text Book of Dynamics, CBS Publishers and distributors, Delhi, 2002 Edition.
5. Loney, S. L.: An Elementary Treatise on The Dynamics of a Particle and of Rigid Bodies, Rahda publishing House, Kolkata, 2002 Edition (only for Unit I).

Reference Book

1. Coddington, Earl A.: An Introduction to Ordinary Differential Equations, PHI Learning Pvt. Ltd., New Delhi, 1998 Edition.
2. Ramsay, A. S.: Dynamics, Part I, Cambridge University Press, 1993 Edition.
3. Singh, K. K. : Text Book of Dynamics, PHI Learning pvt. Ltd., New Delhi, 2011.