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H 61

Advanced Calculus

(100 marks, 80 lectures)

(To answer five questions, choosing one out of two questions from each unit)

UNIT I: Riemann integral of functions of one variable; Darboux's theorem (statement and application); conditions for integrability; classes of bounded and integrable functions; properties of integrable functions; inequalities for integrals; functions defined by integrals; their continuity and differentiability; Mean value theorems for integrals.
Improper integrals; test for convergence when the integrand is non-negative; absolute convergence; tests for absolute and conditional convergence, beta and gamma functions; Abel's theorem, Dirichlet's theorem; Frullani's integral.

UNIT II: Integrals as functions of parameters; continuity, differentiability and integrability of such a function; applications to evaluation of integrals
Improper integrals as functions of a parameter; uniform convergence and tests for uniform convergence; continuity, differentiability and integrability of uniformly convergent improper integrals of continuous functions involving parameters; evaluation of integrals;

UNIT III: Line integral in R^2; Riemann integral of real valued functions of two variables; evaluation of double integrals – change of order of integration; change of variable and simple problems; Green's theorem in R^2, Surface Integral and Stokes Theorem, Volume integral and Gauss's divergence theorem (statements and applications only)

UNIT IV: Basic properties of Euclidean distance function in R^n; neighbourhoods, open sets, closed sets, limit points, interior points in R^n $(n = 1, 2, 3)$; Bolzano-Weierstrass theorem; Cantor intersection theorem (nested interval) ; Lindelof covering theorem.
Compact sets; Heine-Borel theorem; equivalent statements; study of maps from subsets of $R^n \rightarrow R^m$ $(m, n = 1, 2, 3)$: continuity, in terms of ϵ-δ notation; in terms of inverse images of open and closed sets; elementary properties of continuous functions; continuous functions on compact sets; special cases of continuous real valued functions on closed, bounded intervals of R: bounds.

UNIT V: Intermediate value theorem; uniform continuity; discontinuities of real valued functions; monotonic functions; continuity of the inverse of a strictly monotonic function.
R^m -valued functions of two or three variables $(m = 1, 2, 3)$; partial derivatives; directional derivatives; total derivative, Jacobian; change in the order of partial derivatives, statements of Young's Theorem, Schwarz's Theorem and their applications, differentiation of composite functions; chain rule.

BOOKS

Text Books:

1. Narayan, S.: A Course of Mathematical Analysis, S. Chand. Delhi, 2003 Edition.
2. Apostol, Tom A. : Mathematical Analysis, Narosa Publishing House, 2002 Edition.

Reference Books

1. Stewart, J.: Essential Calculus Early Transcendentals, Thomson Brooks/Cole, USA, 2007 Edition.

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2. Bartle, R. G., and Sherbert, D. R. : Introduction to Real Analysis, John Wiley & Sons, Inc, 2000 Edition.
3. Rudin, W. : Principles of Mathematical Analysis, Mc Graw–Hill Publications, 1976 International Editions (Reprint 1996).
4. Malik, S. C. and Arora, S. : Mathematical Analysis, New Age International (P) Ltd., 1992 Edition (Reprint 2001).
5. Ghosh, R.K. and Maity, K.C.: Introduction to Analysis, New Central Book Agency (P) Ltd, 2002 Edition.

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